

Harnessing Water Power on a Small Scale

Volume 2

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## Hydraulics Engineering Manual

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**Supplement:**

**Selected  
Nomograms and Diagrams**

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## 1. Discharge Nomogram of STRICKLER

This nomogram serves to estimate the discharge in a river or a canal.  
The nomogram is based on the formula:

$$\{ 2.1 \} \quad Q = k s^{1/2} \frac{A^{5/3}}{U^{2/3}}$$

where:  
 $k$  = friction coefficient [ - ]  
 $s$  = slope [ - ]  
 $A$  = wetted, cross sectional area [  $m^2$  ]  
 $U$  = wetted perimeter [  $m$  ]  
 $Q$  = river discharge [  $m^3/s$  ]

STRICKLER also established a formula for the determination of the friction coefficient  $k$ :

$$\{ 2.2 \} \quad k = \frac{21.1}{d^{1/6}}$$

where:  $d$  = size of gravel or boulders [  $m$  ]

For values of  $k$  refer to the table in Figure 2.1:

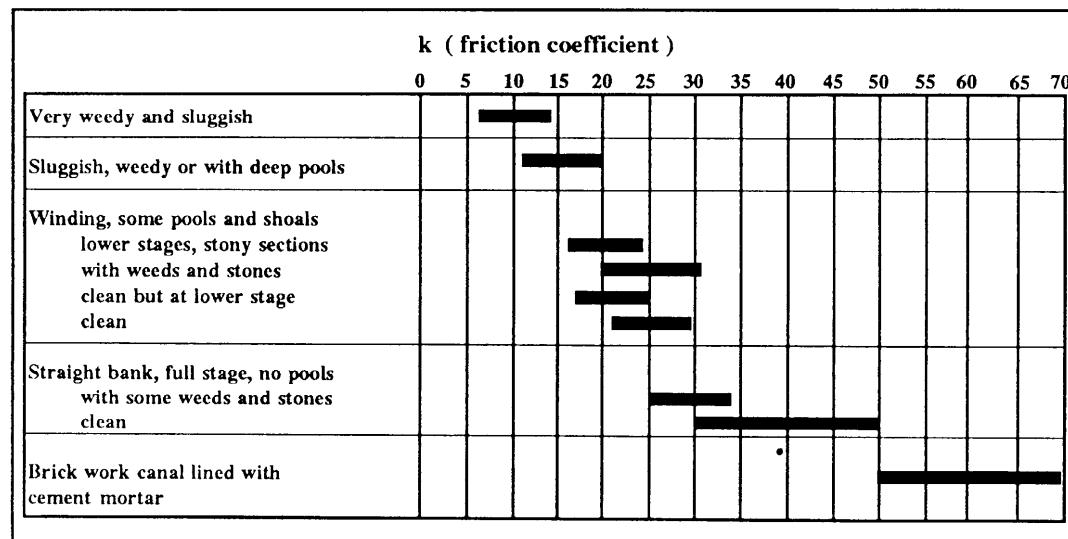
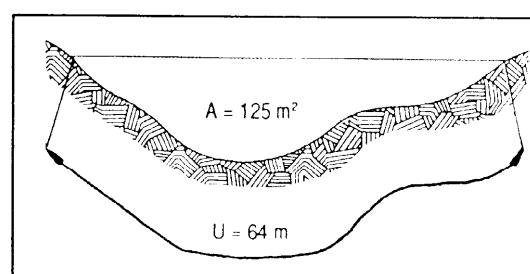


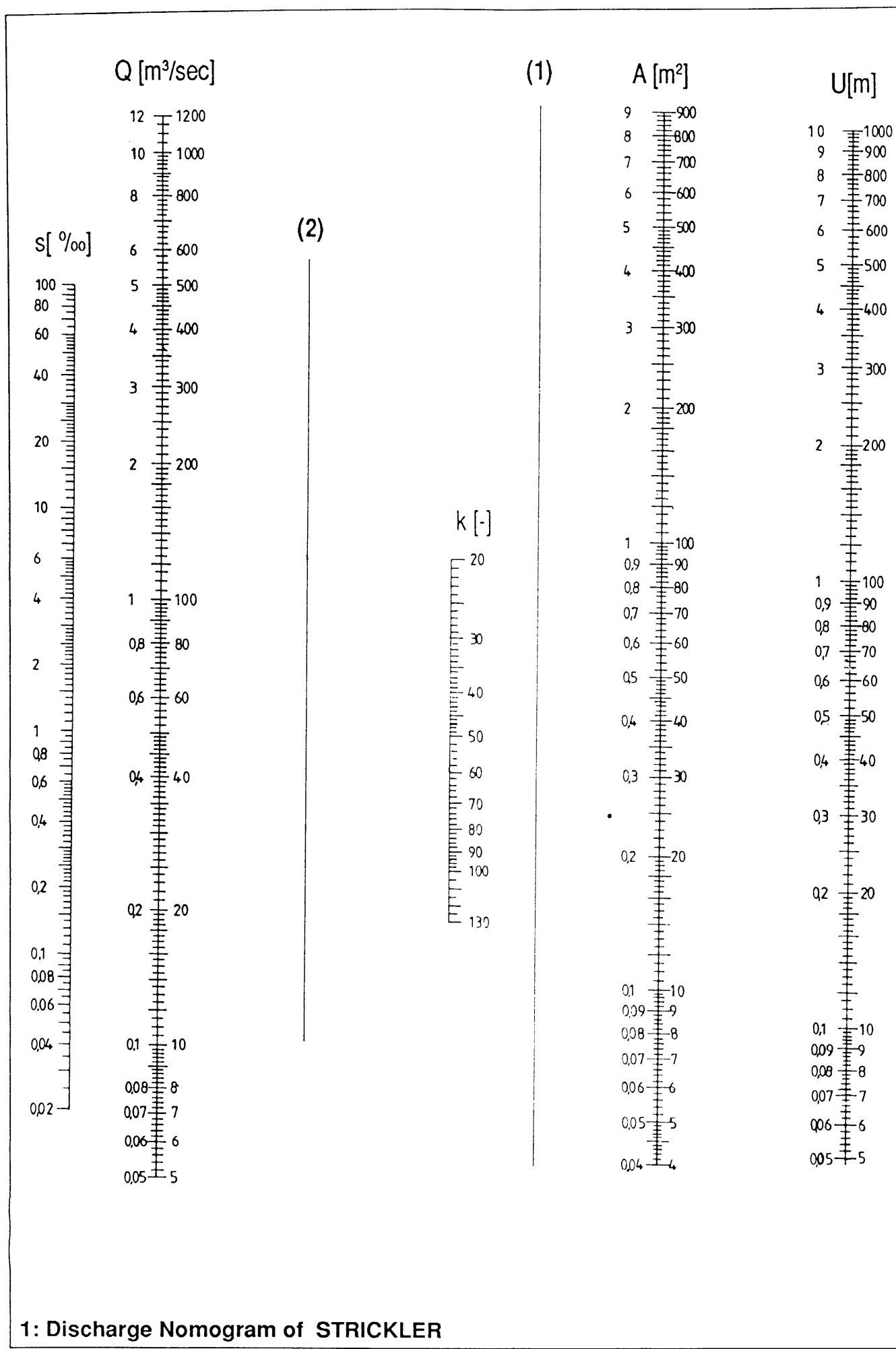
Fig. 2.1: Friction coefficients  $k$  of STRICKLER

### Example of how to use the nomogram

A sluggish, weedy river bed with a few stony sections and a slope of  $1\text{‰}$  shows a wetted cross sectional area of  $125\text{ m}^2$  and a wetted perimeter of  $64\text{ m}$ .

$$\left. \begin{array}{l} k = 30 \\ s = 1\text{‰} \\ A = 125\text{ m}^2 \\ U = 64\text{ m} \end{array} \right\} \Leftrightarrow Q = 185\text{ m}^3/\text{s}$$





## 2. Nomogram for flow in rectangular canal cross sections

Channels with rectangular cross-sections, which implies vertical side walls, are commonly used where it is for some reason not convenient to build channels with a hydraulically better trapezoidal cross-section. The main advantage of rectangular canals is relative ease of construction and a smaller width required for the same flow as compared to trapezoidal canals. Disadvantages chiefly are higher structural stress due to pressure on the side walls and a higher risk of caving in if used in unlined earth construction.

There are four parameters which influence flow in rectangular cross-sections, identical to the relation of STRICKLER (2.1) valid for any cross-section:

$$(2.1) \quad Q = k s^{1/2} \frac{A^{5/3}}{U^{2/3}} \quad [\text{m}^3/\text{s}]$$

$A$  = cross-sectional area [ $\text{m}^2$ ]  
 $U$  = wetted perimeter [m]

where:  
 $k$  = friction coefficient of STRICKLER [-]  
(refer to the note at the top of page 33)  
 $s$  = the slope or gradient of the channel [-]  
 $A$  = cross-sectional area [ $\text{m}^2$ ]  
 $U$  = wetted perimeter [m]

may be expressed by the relation of the channel

For the special case of rectangular cross-sections,  $A$  and  $U$  may be expressed by the relations

resulting in equation (2.5):

$$(2.5) \quad Q = k s^{1/2} \frac{(B h)^{5/3}}{(B + 2h)^{2/3}} \quad [\text{m}^3/\text{s}]$$

parameters  $s$ ,  $k$ ,  $B$ , and the relation  $h/B$ . If any one of the missing parameter, as shown in the

resulting in equation (2.5):

$$(2.5) \quad Q = k s^{1/2} \frac{(B h)^{5/3}}{(B + 2h)^{2/3}} \quad [\text{m}^3/\text{s}]$$

The nomogram contains the flow  $Q$  and the four parameters  $s$ ,  $k$ ,  $B$ , and  $h/B$ . If any four of the five are known, it is possible to find the solution for the fifth. For example,

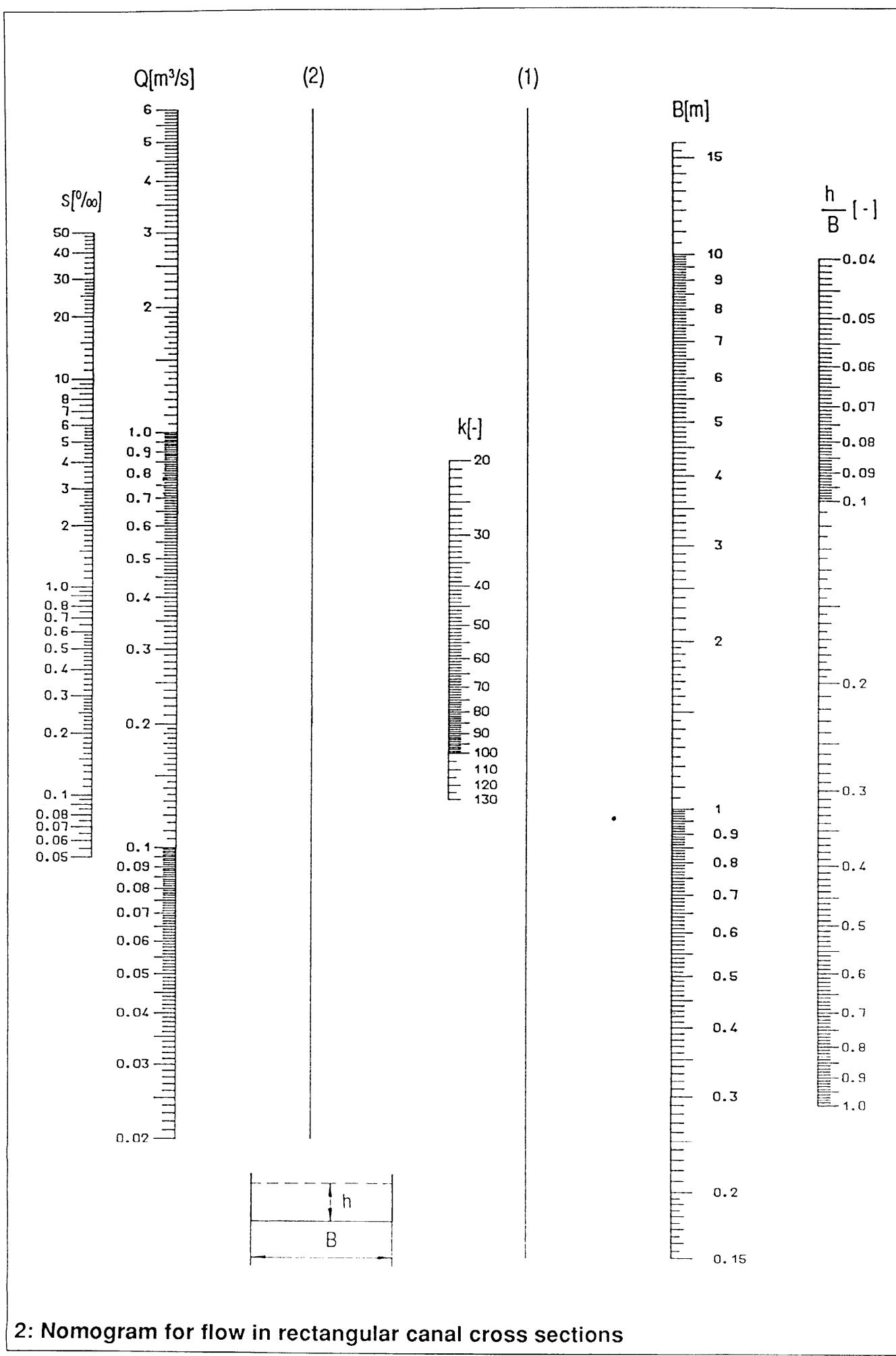
and  $B = 0.5$  [m].

to 70 on the  $k$ -scale, thereby establishing an intersection point on reference line (2). Complete the task by tracing a line from the value of 0.5 on the  $B$ -scale and upto the scale  $h/B$ . The value 0.5, gives  $h/B = 0.50$  [m] for the resulting water

depth  $h$ .

Example: find the water depth  $h$  if  $Q = 0.22$  [ $\text{m}^3/\text{s}$ ],  $k = 70$ ,  $s = 10\%$  and  $B = 0.5$  [m].

Solution: trace a line from the value 0.22 on the  $Q$ -scale through the intersection point previously found, upto reference line (1). The intersection point on reference line (1) through the value of 70 on the  $k$ -scale gives  $h/B = 0.50$ , and multiplying this value by  $B$  gives the water depth.



2: Nomogram for flow in rectangular canal cross sections

### 3. Nomogram for optimal channel sections

What means hydraulically well designed ?

**Definition:** Among all possible channel cross sections having an equal cross-sectional area, the same surface roughness and the same channel slope, the one having the highest water discharge per unit of time is hydraulically the best.

This means when choosing a suitable channel cross section that we are aiming at a maximum flow velocity. This is the case, if the wetted perimeter is a minimum.

The flow velocity can be expressed by the formula :

$$\{2.6\} \quad v = k s^{1/2} \frac{A^{2/3}}{U^{2/3}}$$

v = flow velocity [m/s]

s = slope [ - ]

A = cross sectional area [ $m^2$ ]

U = wetted perimeter [ m ]

Among all curves, the circle encloses the biggest area for a given perimeter. Therefore the circle is the hydraulically best shape for closed conduits, the semi-circle the best for open channels.

As rectangular and trapezoidal channel cross sections are common, it is of interest to know the optimal dimensions for such channel cross sections. For rectangular cross sections, the optimal width and depth are found if the following relation is observed:

$$\{2.7\} \quad b_{opt} = 2 h$$

b = channel bottom width [m]

h = channel depth [m]

channel width and side wall slope to channel depth

= water depth [ m ]

n =  $\tan \alpha$  = tangent of side wall slope  
 $\alpha$  = n

, which means  $n = \angle \alpha$  therefore angle =  $\arctan(1/n)$

obtain the hydraulically best possible channel

Solution :

use with the point 2 on the a-scale and point

42 m

$$\sqrt{1 + \cot^2 \alpha} - \cot \alpha$$

: n =  $\tan \alpha = \frac{\text{tangent of side wall slope}}{\sqrt{2 - \cos \alpha}}$

For trapezoidal cross sections, the optimal relation of channel width and side wall slope to channel depth is somewhat more complex and needs to satisfy the formula

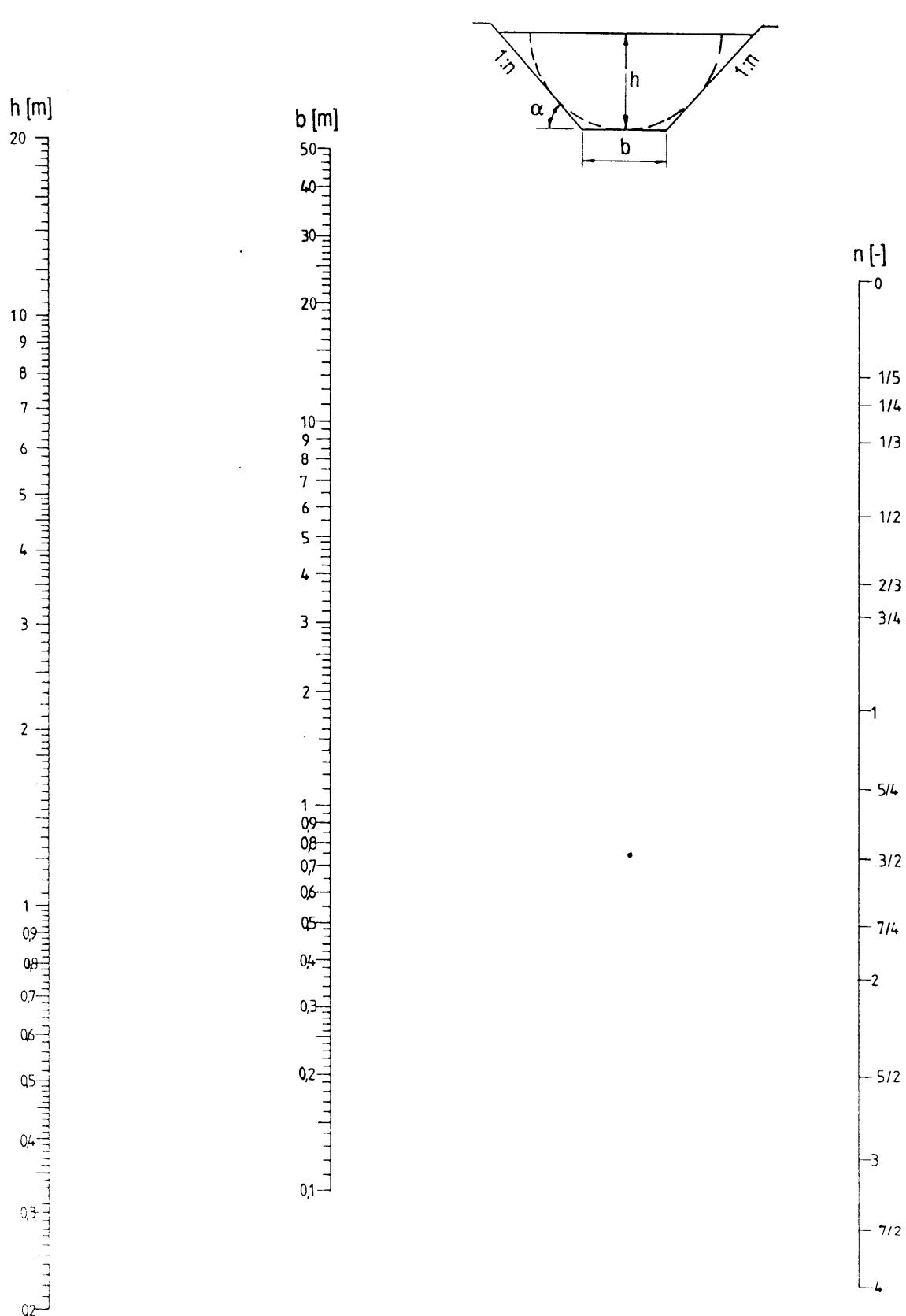
$$\frac{1}{2} \sqrt{\frac{1}{1 + \cot^2 \alpha} - \cot \alpha}$$

#### Example

The slope of the channel side wall has been chosen to be 1 : 2 (angle =  $26,56^\circ$  ( $\arctan 0,5$ )). The water depth shall be 3 m ( $n = 3$  m). What is the optimal channel bottom width ( $b = ?$ ), in order to obtain the hydraulically best possible channel cross section ?

Solution :

Use the nomogram and connect the point 2 [ m ] on the b-scale with the point 3 [ m ] on the a-scale and draw a line through the value of the intersection point on the b-scale :  $\Rightarrow b = 1,4$  m



3: Nomogram for optimal channel sections

#### 4. Nomogram for trapezoidal channel cross sections

This nomogram is very versatile. It shows the relationship of discharge, flow velocity and the geometrical shape of trapezoidal channel cross-sections. It is preferably used in conjunction with the discharge nomogram of STRICKLER in section 2.2.1, which is for the pre-selection of the slope of a channel in order to obtain a certain discharge with a given cross-sectional area, or vice versa. When using the nomogram, reference should also be made to the nomogram for hydraulically well designed shapes for trapezoidal channels, in section 2.2.3. The nomogram in fig. 2.6 is based on the following formulae:

{2.9}

$$Q = v \cdot A$$

where:

$Q$  = discharge [ $\text{m}^3/\text{s}$ ],  
 $v$  = flow velocity [ $\text{m}/\text{s}$ ],  
 $A$  = cross-sectional area [ $\text{m}^2$ ]

{2.10}

$$A = b h + h^2 n$$

where:

$b$  = channel bottom width [ $\text{m}$ ],  
 $h$  = water depth [ $\text{m}$ ],  
 $n$  = cotangent of the side wall slope [-]

{2.11}

$$h = \frac{1}{2} \left[ \sqrt{\left( \frac{b}{n} \right)^2 + \frac{4A}{n}} - \frac{b}{n} \right]$$

##### Example:

The task is to design a trapezoidal channel having the following properties:

- design discharge  $Q = 1.335 \text{ [m}^3/\text{s]}$ ,
- flow velocity in the channel  $v = 1.5 \text{ [m}/\text{s}]$
- cotangent  $\alpha$  of the side wall slope  $n = 2 \text{ [-]}$
- friction coefficient  $k = 30 \text{ [-]}$

Solution:

Step 1: find the cross-sectional area  $A \text{ [m}^2]$  in the nomogram in fig. 2.6: trace a line from the  $\frac{Q}{n}$ -scale at the value of  $\frac{1.335}{2} = 0.668$ , to the value of  $1.5 \text{ [m}/\text{s}]$  on the  $v$ -scale and extending the line upto the  $\frac{A}{n}$ -scale. The result reads 0.445, and as  $n = 2 \Rightarrow A = 2 \cdot 0.445 = 0.890 \text{ [m}^2]$ .

Step 2: find the optimal water depth  $h \text{ [m]}$  for hydraulically well designed shapes of trapezoidal channels: since the bottom width  $b$  is not known for the moment, none of the nomograms can be used directly. The depth  $h$  must be calculated by the formula:

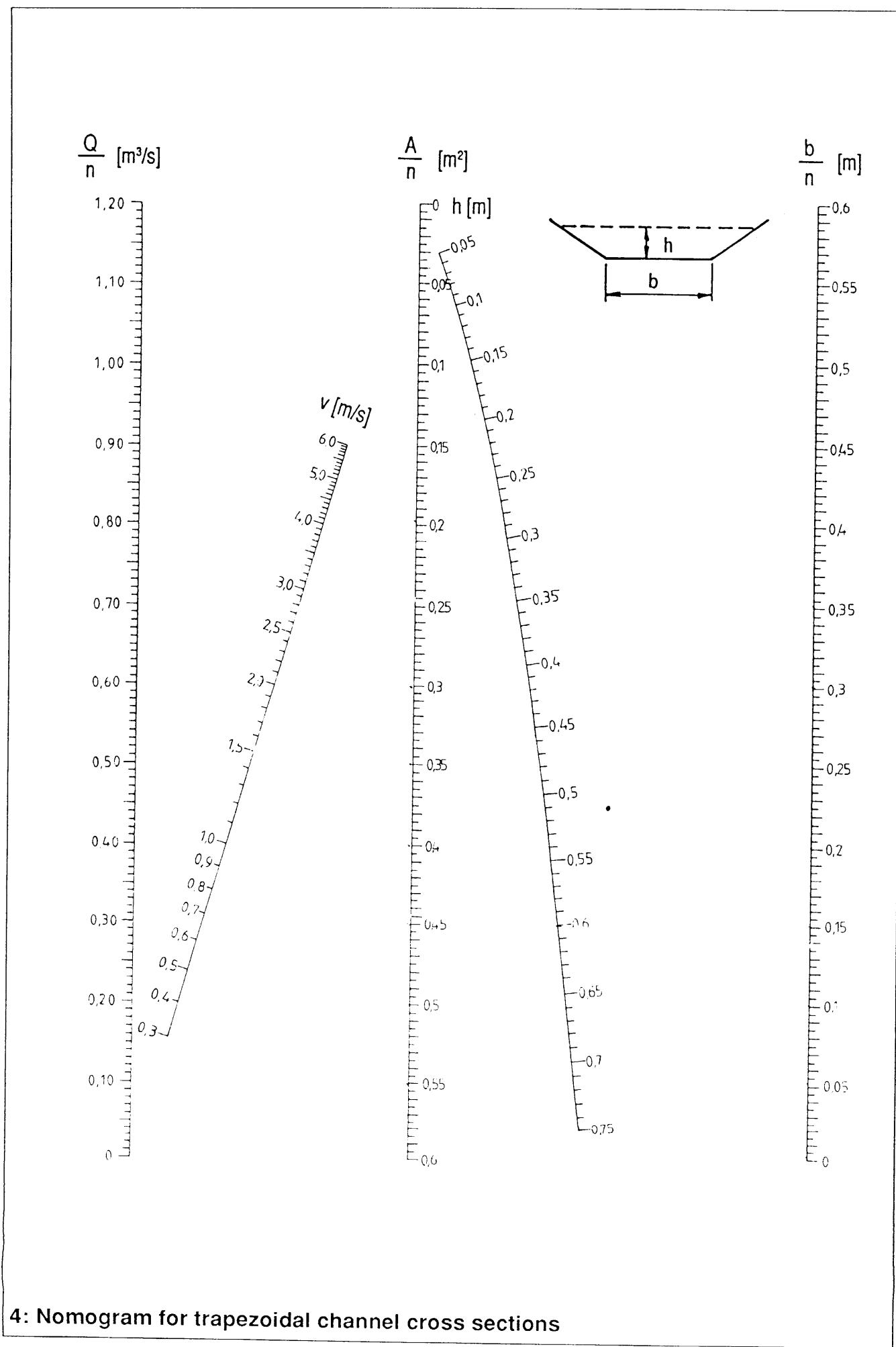
{2.12}

$$h = \sqrt{\frac{A \sin \alpha}{2 - \cos \alpha}}$$

where:  $\alpha$  is the side wall angle of the channel and as the cotangent  $\alpha = 2$ , angle  $\alpha = 26.565^\circ$

$$h = \sqrt{\frac{0.89 \cdot \sin 26.565}{2 - \cos 26.565}} = 0.6 \text{ [m]}$$

Step 3: find the canal bottom width  $b \text{ [m]}$ : by using the nomogram, trace a line starting on the  $\frac{A}{n}$ -scale at the value of 0.445, intersecting the  $h$ -scale at the value of 0.6 and extending upto the  $\frac{b}{n}$ -scale, where the value of 0.1415 is found. Since  $n = 2 \Rightarrow b = 0.1415 \cdot 2 = 0.283 \text{ [m]}$ .



## 5. Nomogram for discharge measurement with weirs

The nomogram is based on the discharge formula :

$$(2.15) \quad Q = \frac{2}{3} \mu b h \sqrt{2gh}$$

where:  
**Q** = discharge [ $\text{m}^3/\text{s}$ ],  
**b** = crest width [m]  
**h** = piezometric head of the upstream  
water surface above crest level [m]  
**g** = gravitational constant = 9.81 [ $\text{m}/\text{s}^2$ ]  
 **$\mu$**  = discharge coefficient [m]

For the case of a full-width weir where  $B = b$ , BUNTSCHU has given a value of approximation for the discharge coefficient  $\mu$  as:

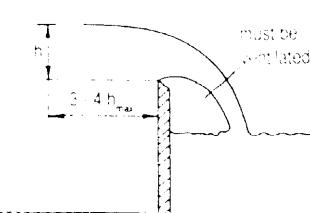
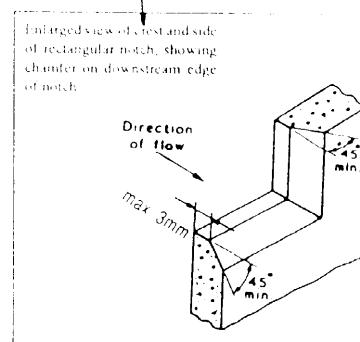
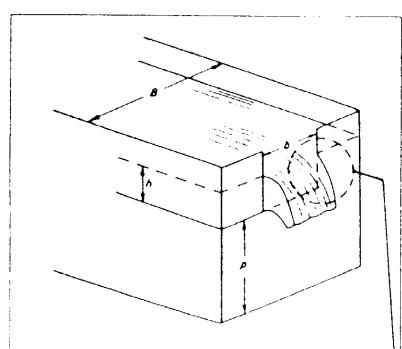
$$(2.16) \quad \mu = \sqrt{\frac{1}{3}} = 0.577$$

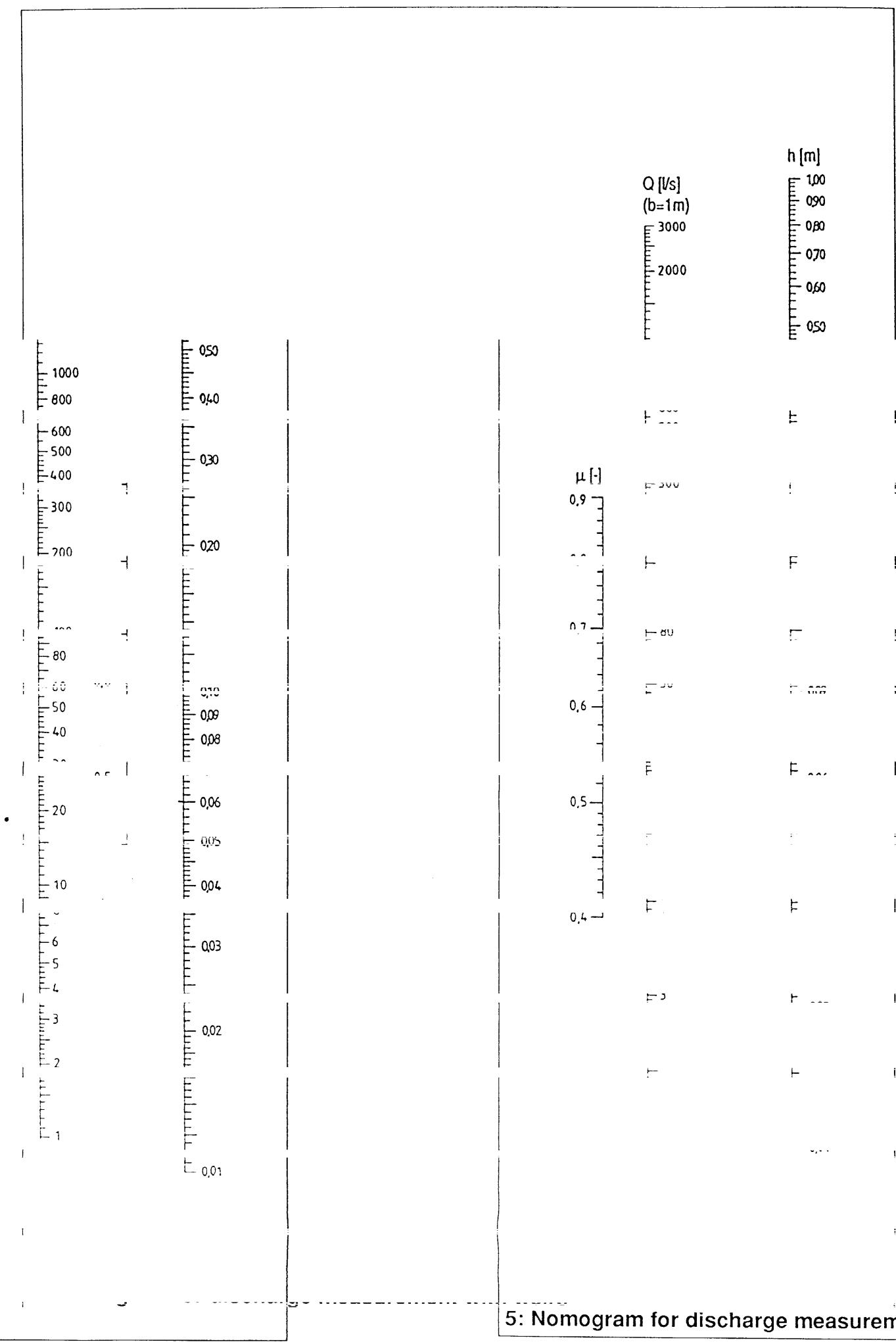
The nomogram is valid for weirs of  $b = 1.0$  [m], either full-width with the value of  $\mu$  as in equation (2.16) or with different values of  $\mu$  for partially contracted weirs. The latter are recommended for accurate measurements and  $\mu$  must be calculated with equation (2.17), which has been specified by the Swiss Association of Standards:

$$(2.17) \quad \mu = \left\{ 0.578 + 0.037 \left( \frac{b}{B} \right)^2 + \frac{0.00365 - 0.0030 \left( \frac{b}{B} \right)^2}{h + 0.0016} \right\} \cdot \left\{ 1 + 0.5 \left( \frac{b}{B} \right) \left( \frac{h}{h + P} \right)^2 \right\}$$

where:  
**b** = crest width [m]  
**B** = channel width [m]  
**h** = piezometric head [m]  
**P** = weir height [m]

and the following limitations are strictly observed:  
-  $h/P$  shall not exceed 1.0  
- the head  $h$  shall exceed  $0.025 \cdot b/B$  [m], but shall be less than 0.8 [m] according to fig. 2.7  
-  $b/B$  shall not be less than 0.3  
- the weir height  $P$  shall be at least 0.3 [m]





## 6. Nomogram for head losses in penstock

For successful micro hydropower projects, it is essential to know head losses  $\Delta H$  due to friction in the penstock as well as elsewhere in the waterway at an early stage because the actual turbine working head  $H_{net}$  is a key parameter in turbine selection and design, affecting the operating speed, discharge through the turbine and power output.

The actual working head or net head  $H_{net}$  may be calculated by subtracting total head losses  $\Delta H_{tot}$  from the geodetic head  $H$ , where  $\Delta H_{tot}$  consists of total losses due to friction in the penstock plus all other losses applying.

$$\{2.18\} \quad H_{net} = H - \Delta H_{tot} \text{ [m]}$$

Head losses in the penstock  $H$  may be calculated by equation {2.19} :

$$\{2.19\} \quad \Delta H = \frac{4^{10/3}}{\pi^2} \frac{Q^2 L}{k^2 D^{16/3}} \text{ [m]}$$

where:  $\pi = 3.141593$

$Q$  = discharge through the penstock [m<sup>3</sup>/s]

$L$  = straight length of the penstock [m]

$D$  = the inside diameter of the penstock [m]

$k$  = the friction coefficient used [-].

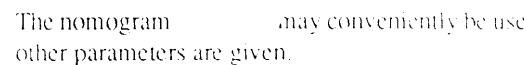
according to STRICKLER

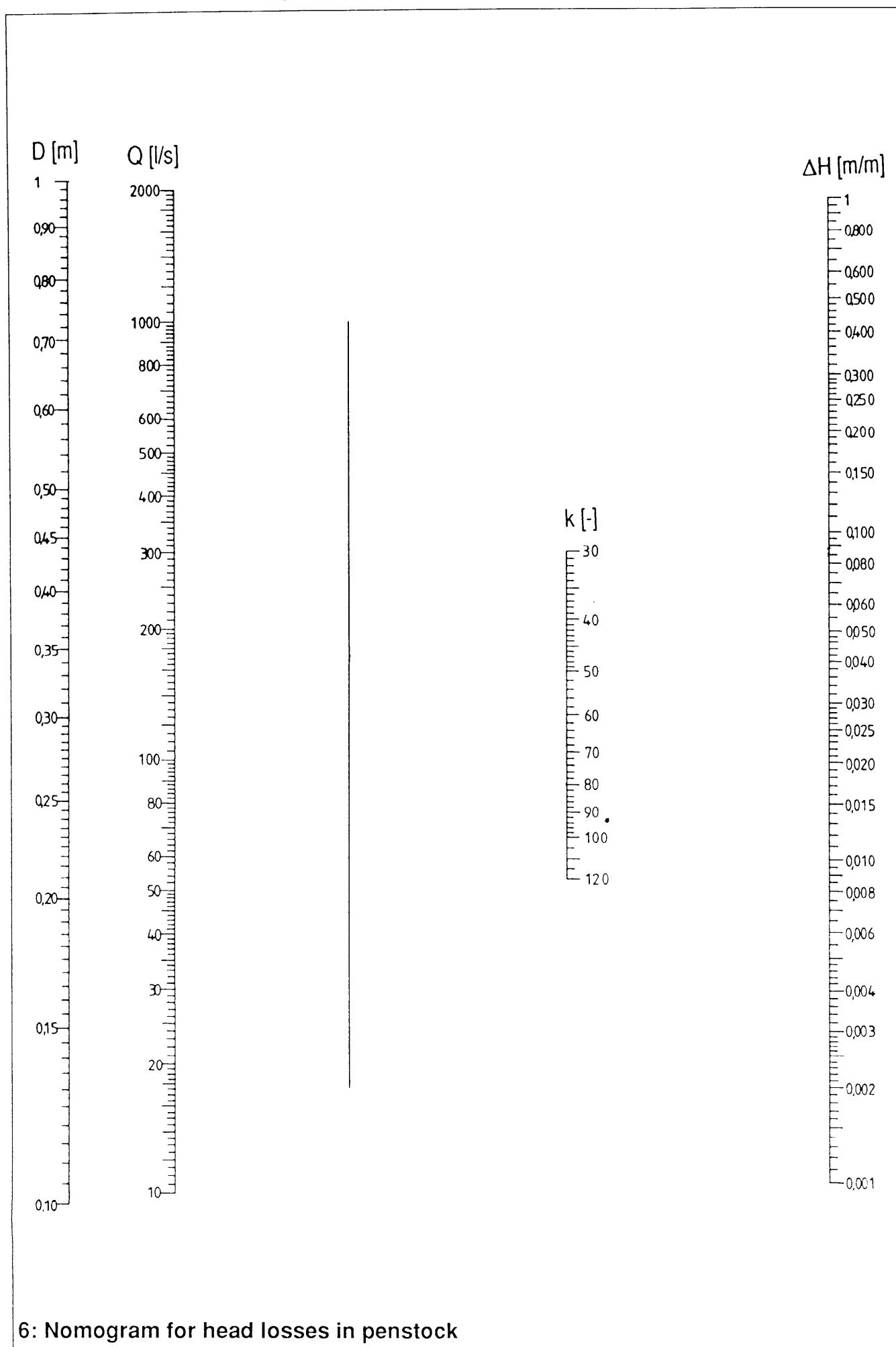
For convenience, the formula may also be written as:

$$\{2.20\} \quad \Delta H = 10.294 \frac{Q^2 L}{k^2 D^{5.33}} \text{ [m]}$$

In order to be able to deduct head losses from gross head as a basis for further calculations, units shown are usually in meters. This, however is not strictly correct. Proper units are mkg/s<sup>2</sup>, which are energy units, which head losses actually represent. The following facts become apparent when looking at equation {2.20}

- The influence of penstock length  $L$  is directly proportional to head losses.
- The influence of discharge  $Q$  is proportional to the second power of  $Q$ , i.e. doubling the discharge results in a four-fold increase of head losses.
- The influence of friction  $k$  is reciprocally proportional to the second power of  $k$ , i.e. double the  $k$ -number results in head losses of 25% of the original value.
- The influence of diameter  $D$  is reciprocally proportional to the power of 5.33 of the diameter, i.e. doubling the diameter results in head losses of  $1/2^{5.33} = 2.5\%$  of the original value. Reducing the diameter to half, increases head losses about fourty times

The nomogram  may conveniently be used to find one of the head loss related values, if the three other parameters are given.



6: Nomogram for head losses in penstock

## 7. Nomogram for minimal penstock thickness

The wall thickness selected for a penstock pipe mainly depends on the following parameters:

- the material selected for the penstock
- the diameter of the pipe
- the operating pressure

When selecting the material for a penstock, it is of interest to know the permissible hoop stress, the ultimate tensile bending stress and stresses due to thermal expansion. To calculate the latter, we need to know YOUNG's Modulus of Elasticity, and of interest is also the question of corrosion and weldability of the material used..

The selection of the pipe diameter depends primarily on the design discharge, the length of the penstock and the acceptable head losses as well as economical considerations.

The operating pressure depends not only on the static head of the installation, but also on transient surge pressure effects. It is understood, that the penstock must withstand the maximum operating pressure occurring.

Each term, the pressure  $p$ , the permissible stress  $\sigma$  and the penstock diameter  $D$  need to be carefully evaluated before using these data for the determination of the wall thickness  $t$ . Further, after having determined the wall thickness, the result needs to be reviewed by incorporating a reasonable safety-factor.

**The nomogram provides minimum values for the wall thickness, not including safety and corrosion allowances. It is based on the hoop-stress formula:**

$$(2.32) \quad t = \frac{p D}{2 \sigma_{\text{perm}}} \quad [\text{cm}]$$

where:

$t$  = wall thickness of penstock pipe in cm  
 $p$  = operating pressure in kgf per  $\text{cm}^2$   
 $D$  = internal diameter of penstock in cm  
 $\sigma_{\text{perm}}$  = permissible stress in kgf per  $\text{cm}^2$

$1 \text{ kgf/cm}^2 = 10 \text{ m of}$   
 $\text{water column}$   
 $1 \text{ kgf} = 9.81 \text{ Newton}$

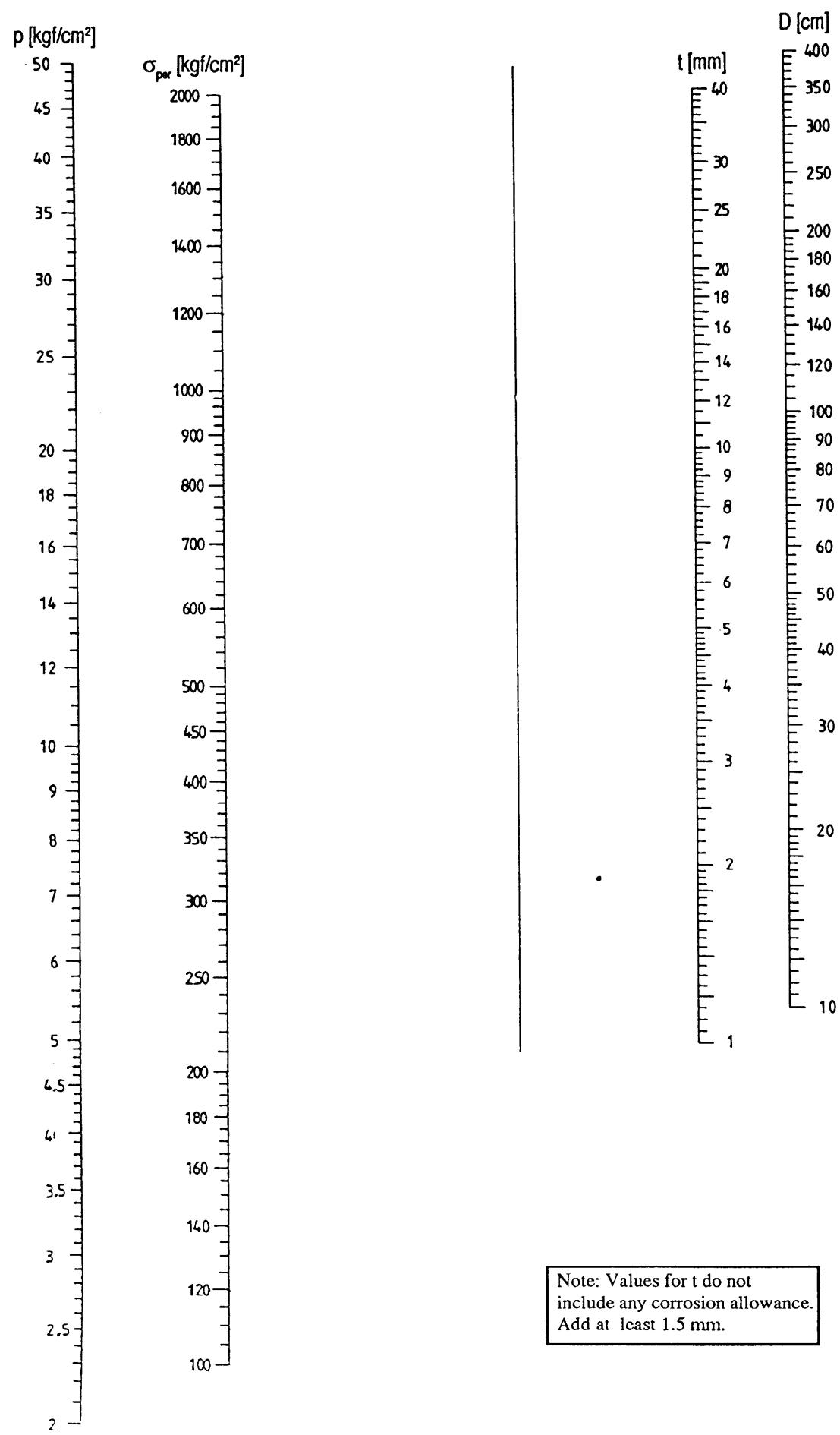
Example:

operating pressure:  $16 \text{ kgf/cm}^2$   
 permissible stress:  $1200 \text{ kgf/cm}^2$   
 internal penstock diamter:  $150 \text{ cm}$

required minimum wall thickness =  $10 \text{ mm}$ ,  
 with a safety factor of  $\frac{3500}{1200} = 2.9$

The table below lists properties of materials used in the manufacture of penstock pipes

Material	YOUNG's modulus of elasticity $E (\text{kgf}/\text{cm}^2)$	Coefficient of linear expansion $a (\text{m}/\text{m} \cdot {}^\circ\text{C})$	Ultimate tensile strength $(\text{kgf}/\text{cm}^2)$	Density $(\text{kgf}/\text{m}^3)$
Steel	$21 \cdot 10^5$	$12 \cdot 10^{-6}$	3500	7900
Polyvinyle chloride (PVC)	$0.28 \cdot 10^5$	$54 \cdot 10^{-6}$	280 *	1400
Polyethylene	$0.02-0.08 \cdot 10^5$	$140 \cdot 10^{-6}$	60 - 90 *	940
Concrete	$2 \cdot 10^5$	$10 \cdot 10^{-6}$		1800-2500
Asbestos cement		$8.1 \cdot 10^{-6}$		1600-2100
Cast iron	$8 \cdot 10^5$	$10 \cdot 10^{-6}$	1400	7200
Ductile iron	$17 \cdot 10^5$	$11 \cdot 10^{-6}$	3500	7300



7: Nomogram for minimal penstock thickness

### 8.1 ALLIEVI chart: pressure rise for uniform gate closure and simple conduits: large $\rho$ and $\theta$ .

Whenever the discharge rate through a penstock is varied, for instance by changing the gate opening at the end of the penstock, the mean flow velocity of the water changes, resulting in fluctuations of pressure.

In case of sudden gate closure at the end of a penstock, the pressure may rise to a significantly higher value. This effect is called water-hammer. Similarly, in case of sudden valve opening, the pressure may drop. The water-hammer effect is related to the conversion of the kinetic energy of the flowing water in the penstock to the work absorbed in stretching the penstock wall and compressing the water column. The theory behind the water-hammer effect is very complex and not easily understood.

The coordinate axes of the ALLIEVI chart are:

the  $\rho$  axis which stands for the penstock parameter  $\rho [-]$  and the  $\theta$  axis which stands for the valve operation parameter  $\theta [-]$ .

The penstock parameter  $\rho$  is expressed by the formula:

$$\{2.33\} \quad \rho = \frac{a v_0}{2 g H_0}$$

where:  $a$  = wave velocity [ m/s ]  
 $v_0$  = flow velocity in the penstock [ m/s ]  
 $g$  = gravitational constant,  $9.81$  [  $\text{m/s}^2$  ]  
 $H_a$  = steady state head [ m ]

The valve operation parameter  $\theta$  is expressed by the formula:

$$\{2.34\} \quad \theta = \frac{a t_c}{2 L}$$

where:  $a$  = wave velocity [ m/s ]  
 $t_c$  = closing or opening time of the valve [ s ]  
 $L_c$  = length of the penstock [ m ]

The **wave velocity**  $a$  which needs to be known for the calculation of  $\rho$  as well as of  $\theta$  is expressed by the formula:

$$\{2.35\} \quad a = \sqrt{1 + \frac{E_w D}{E_t}}$$

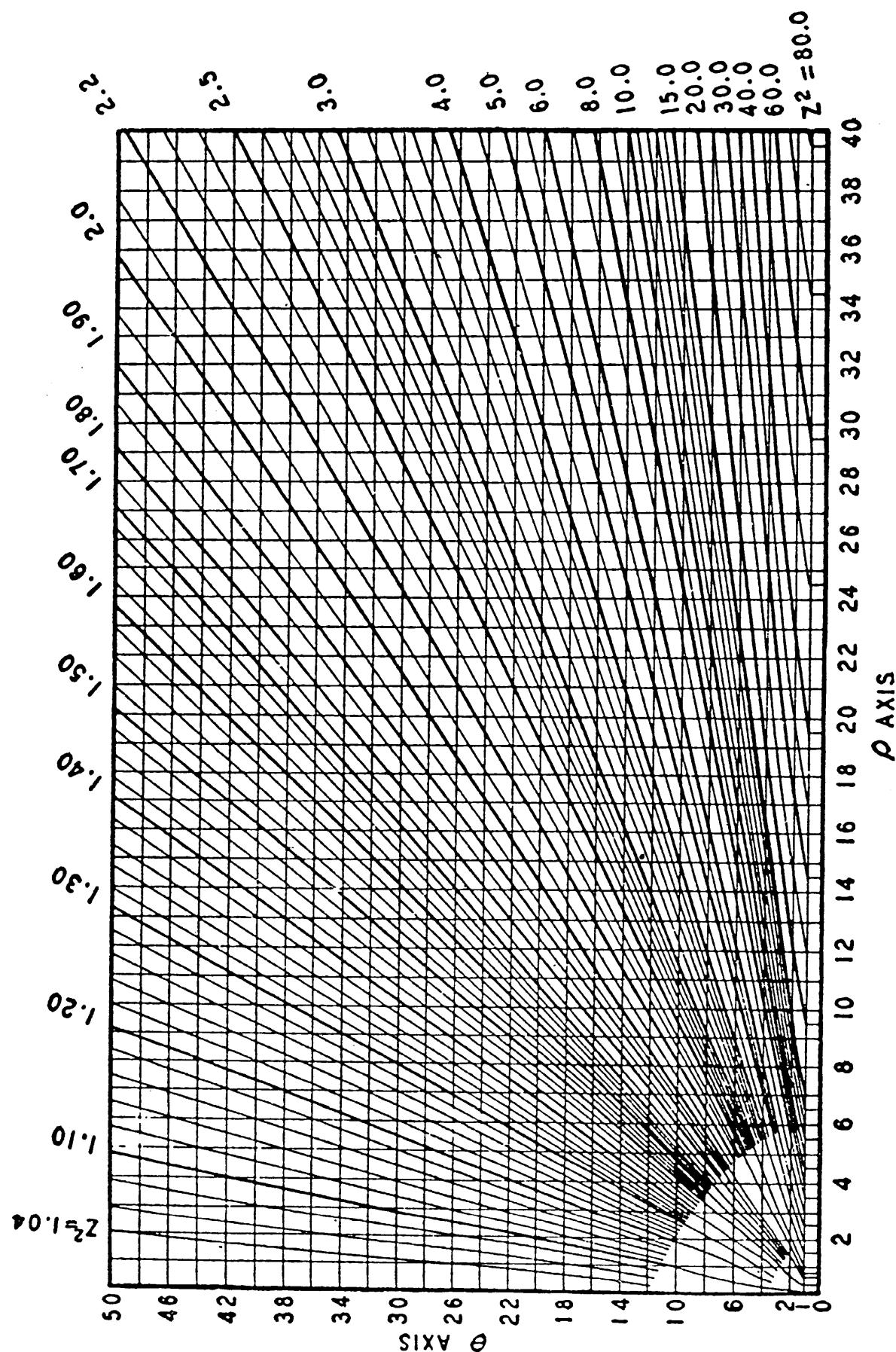
where:  $E_w$  = modulus of elasticity of water = 2030 [ N / mm<sup>2</sup> ]  
 $D$  = internal diameter of penstock [ m ]  
 $E$  = YOUNG's modulus of elasticity of the penstock material [ N / mm<sup>2</sup> ]  
 $t$  = wall thickness of penstock pipe [ m ]

The value  $Z^2$ , which stands for the **pressure rise factor**, may be read from the ALLIEVI chart once values for  $\rho$  and  $\theta$  are determined.

The pressure rise factor is expressed by the formula :

$$\{2.36\} \quad Z^2 = \frac{H}{H_0}$$

where:  $H$  = pressure head [ m ]  
 $H_0$  = steady state pressure [ m ]  
           (net head)



8.1: ALLIEVI chart: pressure rise for uniform gate closure and simple conduits:  
large  $\rho$  and  $\theta$

## 8.2 ALLIEVI chart: pressure rise for uniform gate closure and simple conduits: small $\rho$ and $\theta$ .

### Example

It is planned to use HD polyethylene pipes for a penstock. The permissible pressure for these pipes is not to exceed 10 bar (100 m water column). What is the shortest permissible closing time of the valve at the end of the penstock with the following data:

- net head of the installation	$H_0$	= 67 [ m ]
- length of penstock	$L$	= 200 [ m ]
- internal diameter of penstock pipe	$D$	= 0.1 [ m ]
- wall thickness of penstock pipe	$t$	= 0.01 [ m ]
- flow velocity in the penstock	$v_0$	= 2.5 [ m/s ]
- maximum permissible pressure head	$H_{\max}$	= 100 [ m ]
- penstock material: HD polyethylene	$E$	YOUNG's modulus of elasticity = 1500 [ N/mm <sup>2</sup> ]

Step 1: calculation of pressure rise factor  $Z^2$  [ - ]

$$Z^2 = \frac{H_{\max}}{H_0} = \frac{100}{67} = 1.49$$

Step 2: calculation of wave propagation velocity  $a$  [ m/s ]

$$a = \frac{1425}{\sqrt{1 + \frac{E_w D}{E t}}} \quad \text{where: } E_w = \text{modulus of elasticity of water} = 2030 \text{ [ N/mm}^2\text{ ]}$$

$$a = \frac{1425}{\sqrt{1 + \frac{2030 \cdot 0.1}{1500 \cdot 0.01}}} = 374 \text{ [ m/s ]}$$

Step 3: calculation of penstock parameter  $\rho$  [ - ]

$$\rho = \frac{a v_0}{2 g H_0} \quad \text{where: } g = \text{gravitational constant} = 9.81 \text{ [ m/s}^2\text{ ]}$$

$$\rho = \frac{374 \cdot 2.5}{2 \cdot 9.81 \cdot 67} = 0.71 \text{ [ - ]}$$

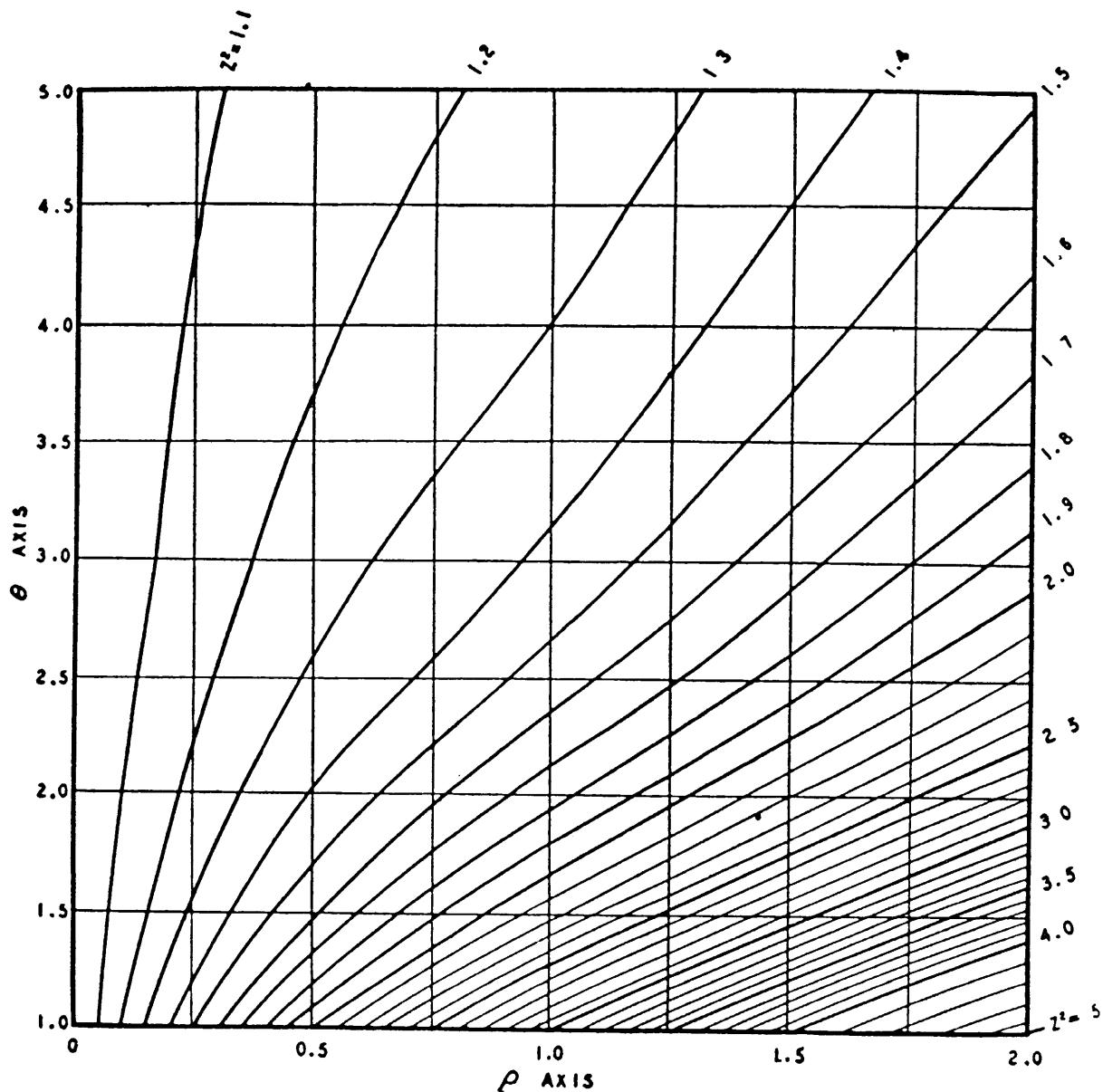
Step 4: determination of the valve operation parameter  $\theta$  [ - ] by means of the ALLIEVI chart:

$$\left. \begin{array}{l} Z^2 = 1.49 \\ \rho = 0.71 \end{array} \right\} \Leftrightarrow \text{ALLIEVI chart (small } \rho \text{ and } \theta \text{ ) } \Leftrightarrow \theta = 2.2$$

Step 5: calculation of shortest permissible closing time  $t_c$  [ s ]

$$t_c = \frac{\theta 2 L}{a} = \frac{2.2 \cdot 2 \cdot 200}{374} = 2.35 \text{ [ s ]}$$

To keep the pressure rise within the limit of  $H_{\max} = 100$  [ m ], the closing time of the gate is not to be less than 2.35 seconds.



8.2 :ALLIEVI chart: pressure rise for uniform gate closure and simple conduits:  
small  $\rho$  and  $\theta$

### 8.3 ALLIEVI chart: pressure rise (medium $\rho$ and $\theta$ ) and wave cycle curves

#### Example

In a scheme with the data given below, the closing time shall be chosen in such a way that the pressure rise does not exceed 20% of the steady state head. Further, it is of interest to find the time after which the wave cycles create the maximum water-hammer.

- net head of installation	$H_0 = 63.9$ [ m ]
- length of penstock	$L = 450$ [ m ]
- internal diameter of penstock pipe	$D = 0.3$ [ m ]
- wall thickness of penstock	$t = 0.01$ [ m ]
- flow velocity in the penstock	$v_0 = 2$ [ m/s ]
- penstock material: mild steel with a YOUNG's modulus of elasticity	$E = 210'000$ [ N/mm <sup>2</sup> ]

Step1: calculation of the wave velocity  $a$  [ m/s ]

$$a = \frac{1425}{\sqrt{1 + \frac{E_w D}{E t}}} \quad \text{where: } E_w = \text{modulus of elasticity of water} = 2030 \text{ [ N/mm}^2\text{ ]}$$

$$a = \frac{1425}{\sqrt{1 + \frac{2030 \cdot 0.3}{210'000 \cdot 0.01}}} = 1255 \text{ [ m/s ]}$$

Step 2: calculation of penstock parameter  $\rho$  [ - ]

$$\rho = \frac{a v_0}{2 g H_0} \quad \text{where: } g = \text{gravitational constant} = 9.81 \text{ [ m/s}^2\text{ ]}$$

$$\rho = \frac{1255 \cdot 2}{2 \cdot 9.81 \cdot 63.9} = 2 \text{ [ - ]}$$

Step 3: calculation of pressure rise factor  $Z^2$  [ - ]: pressure rise = 20 %  $\Leftrightarrow Z^2 = 1.2$  [ - ]

Step 4: determination of the valve operation parameter  $\theta$  [ - ] by means of the ALLIEVI chart.

$$\left. \begin{array}{l} \rho = 2 \\ Z^2 = 1.2 \end{array} \right\} \Leftrightarrow \text{ALLIEVI chart ( medium } \rho \text{ and } \theta \text{ )} \Leftrightarrow \theta = 11 \text{ [ - ]}$$

Step 5: calculation of gate closing time  $t_c$  [ s ]

$$t_c = \frac{\theta L}{a} = \frac{11 \cdot 2 \cdot 450}{1255} = 7.9 \text{ [ s ]}$$

With a gate closing time of 7.9 seconds the pressure rise remains within the limit of 20% of head.

Step 6: determination of the time required by the wave cycles to reach maximum water-hammer.

from the ALLIEVI chart (fig. 2.19) we see that the maximum pressure rise is reached after six wave cycles ( $s_6$ ).

The time for one wave cycle through the entire penstock length is:

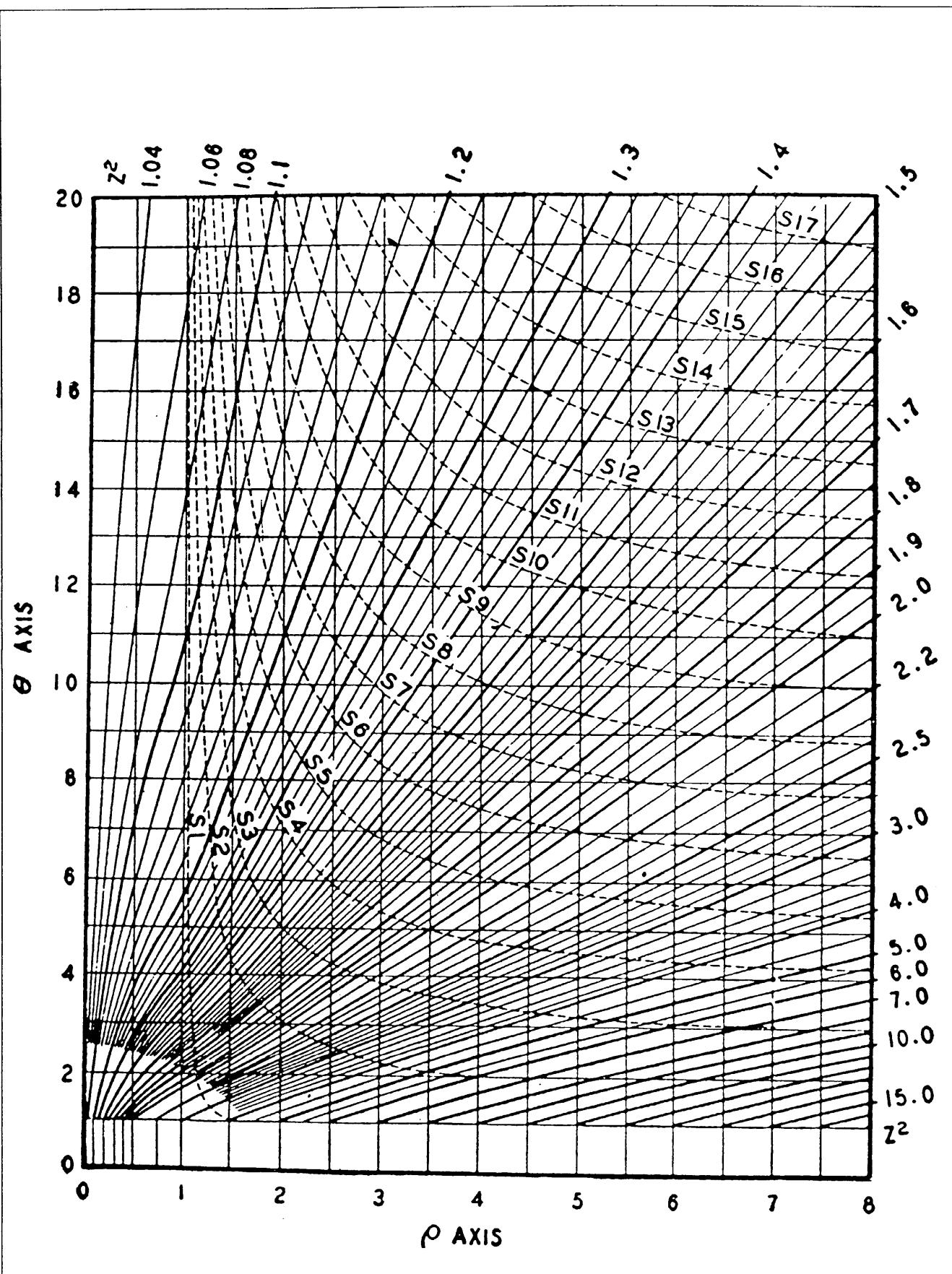
$$\{2.37\} \quad t_{wc} = \frac{4 L}{a} = \frac{4 \cdot 450}{1255} = 1.43 \text{ [ s ]}$$

As six cycles occur to reach maximum water-hammer, the total time required is:

$$t_{tot} = s_6 t$$

$$t_{tot} = 6 \cdot 1.43 = 8.6 \text{ [ s ]}$$

After 8.6 seconds the peak pressure is reached.

8.3 : ALLIEVI chart - pressure rise (medium  $\rho$  and  $\theta$ ) and wave cycle curve

## 8.4 ALLIEVI chart: pressure drop

### Example

In the same way that closing a gate at the end of a penstock initiates a pressure rise, so does opening a gate create a pressure drop. It is therefore of interest to calculate the permissible opening time of a gate for a given pressure drop limit under specific conditions.

-net head of the installation	$H_0$	= 4.4 [ m ]
-length of the penstock	L	= 380 [ m ]
-internal diameter of pipe	D	= 1.1 [ m ]
-wall thickness of penstock pipe	t	= 0.005 [ m ]
-flow velocity in the penstock	$v_0$	= 1.5 [ m/s ]
-penstock material: mild steel	E	= 210'000 [ N/mm <sup>2</sup> ]
YOUNG's modulus of elasticity		
-permissible minimum pressure	$H_{min}$	= 2.2 [ m ]

Step 1: calculation of pressure drop factor  $Z^2$  [ - ]

$$Z^2 = \frac{H_{min}}{H_0}$$

$$Z^2 = \frac{2.2}{4.4} = 0.5$$

Step 2: calculation of wave velocity a [ m/s ]

$$a = \sqrt{\frac{1425}{1 + \frac{E_w d}{E s}}}$$

where  $E_w$  = modulus of elasticity of water  
= 2030 [ N/mm<sup>2</sup> ]

$$a = \sqrt{\frac{1425}{1 + \frac{2030 \cdot 1.1}{210'000 \cdot 0.005}}} = 806 [ m/s ]$$

Step 3: calculation of penstock parameter  $\rho$  [ - ]

$$\rho = \frac{a v_0}{2 g H_0} \quad \text{where: } g = \text{gravitational constant} = 9.81 [ m/s<sup>2</sup> ]$$

$$\rho = \frac{806 \cdot 1.5}{2 \cdot 9.81 \cdot 4.4} = 14$$

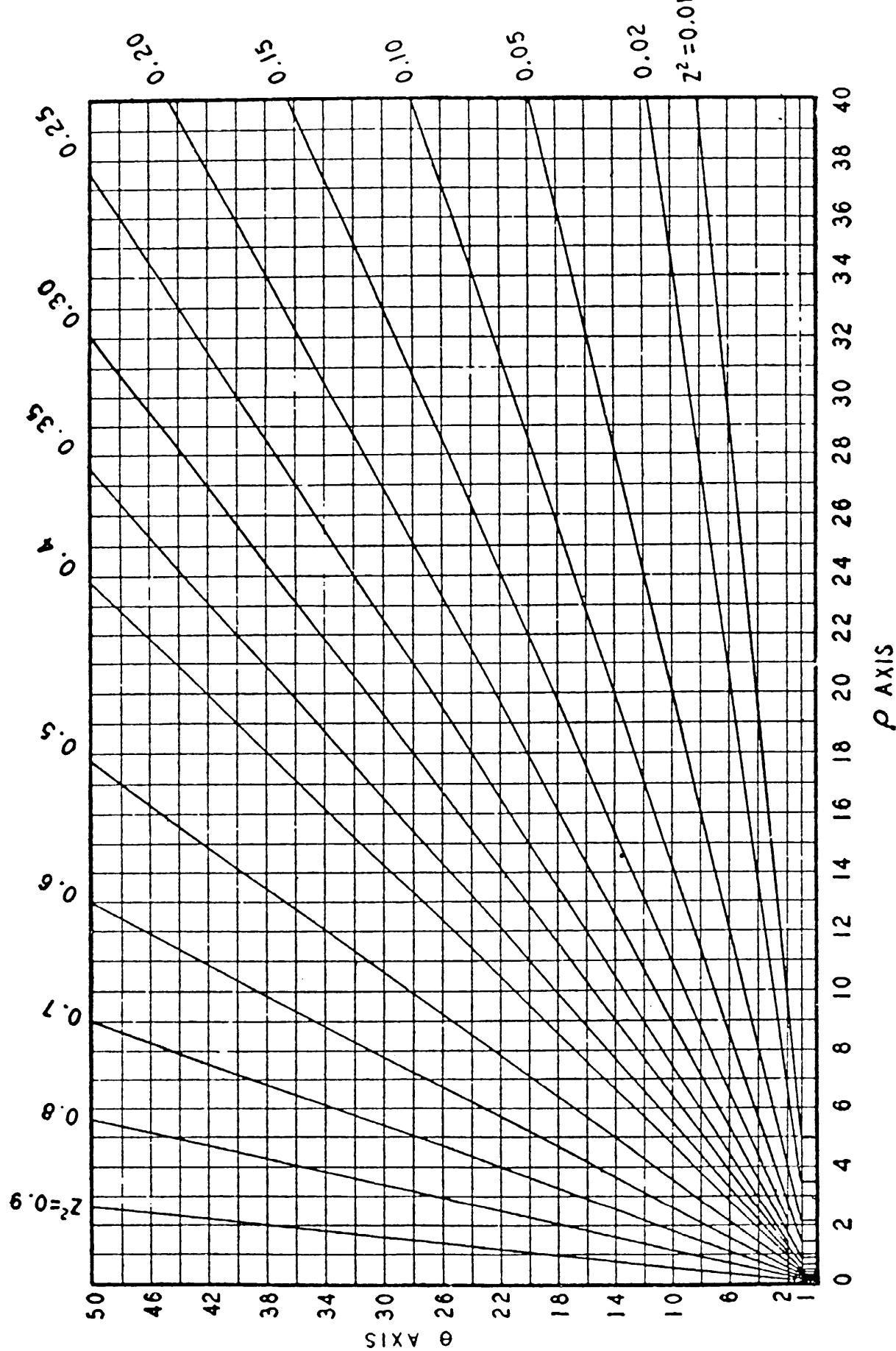
Step 4: determination of the valve operation parameter  $\theta$  [ - ] by means of the ALLIEVI chart, fig. 2.20

$$\left. \begin{array}{l} Z^2 = 0.5 \\ \rho = 14 \end{array} \right\} \Leftrightarrow \text{ALLIEVI chart} \Leftrightarrow \theta = 40$$

Step 5: calculation of shortest permissible opening time  $t_c$  [ s ]

$$t_c = \frac{\theta \cdot 2 \cdot L}{a} = \frac{40 \cdot 2 \cdot 380}{806} = 37.7 [ s ]$$

If the pressure is not to drop below 2.2 [ m ] the opening time of the gate is to be at least 37.7 seconds.



8.4.: ALLIEVI chart: pressure drop

## 9. Diagram of specific speeds

In most cases the expression of the specific speed  $n_s$  helps to make a sensible choice among different types of turbines such as Pelton, Francis, Propeller or Cross Flow.

The specific speed  $n_s$  is a term used to classify turbines on the basis of their performance and dimensional proportions, regardless of their actual size or the speed at which they operate. The specific speed is the speed expressed in revolutions per minute (rpm) of an imaginary turbine, geometrically similar in every respect to the actual turbine under consideration, and capable of lifting 75 kg of water per second to a height of 1 m (effective output 1 metric HP). The mathematical formula for calculating the specific speed reads:

$$\{2.38\} \quad n_s = 3.65 \frac{\sqrt{Q}}{H^{3/4}}$$

where Q is to be inserted in  $m^3/s$  and H in m and n (turbine speed) in rpm .

The same formula may be written as:

$$\{2.39\} \quad n_s = \frac{n \sqrt{P}}{H^{5/4}}$$

where P is the theoretical turbine output in HP ( $P = \frac{Q H}{75}$ )

Note: Above formulae for  $n_s$  are classical and still widely in use, however as they base on metric Horse Power, they do not comply with the SI-units.

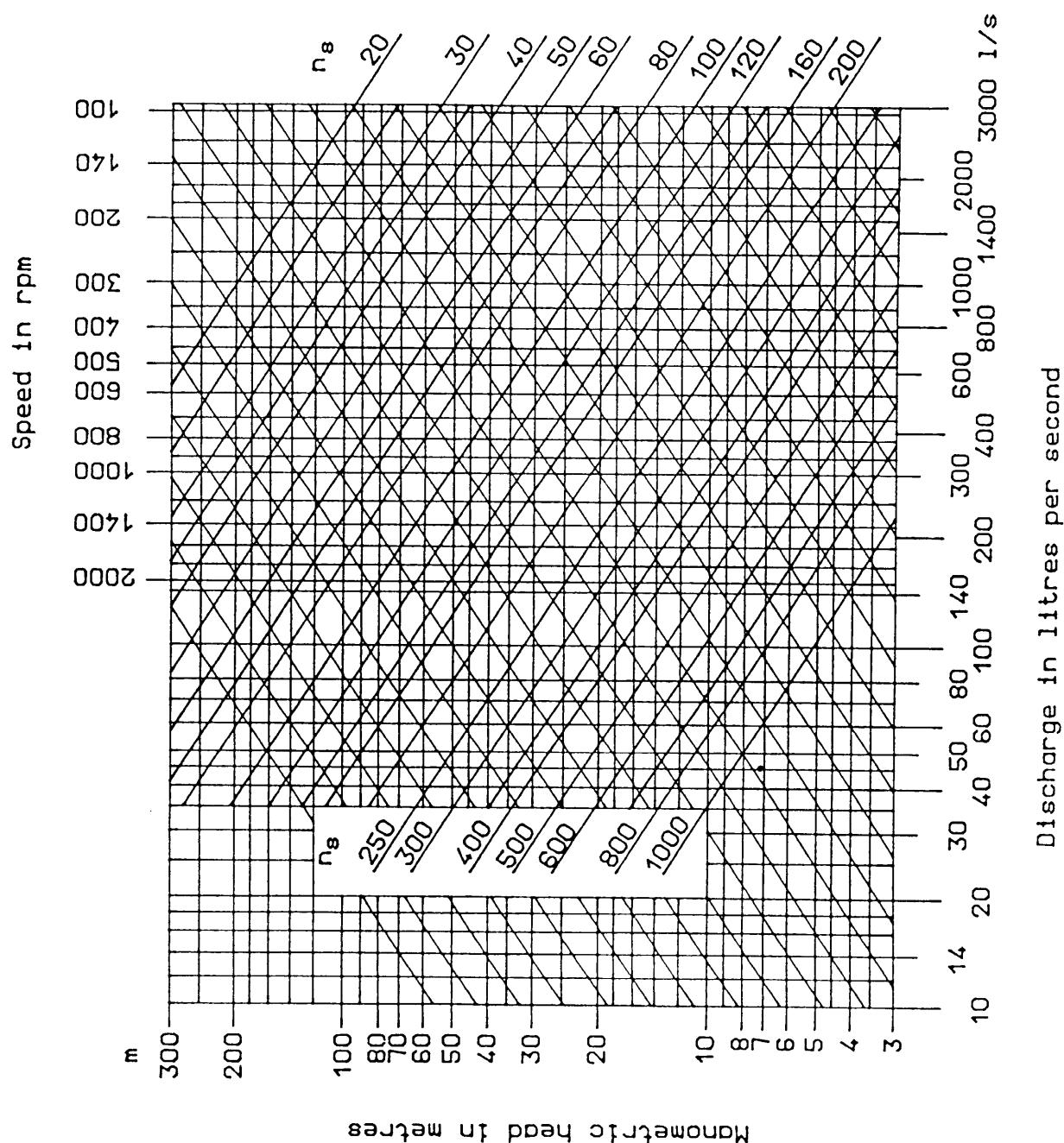
If SI-units are applied, the conversion factor of 1.36 (HP/kW) is to be considered as otherwise the obtained  $n_s$ -value would be 14 % smaller. Hence, for calculating  $n_s$  based on HP but using power in kW, the following formula is derived:

$$n_s = n H^{-5/4} (1.36 P_{[kW]})^{1/2} = 1.166 n H^{-5/4} P_{[kW]}^{1/2}$$

True metric specific speed is denoted  $N_s$  and is getting more and more popular. However, the diagram is based on HP-related  $n_s$ .

For a specific application where operating head, flow and a preferred speed of the turbine are known, the specific speed may be found in the diagram . For a given task the specific speed of a turbine can easily be determined in the same diagram.

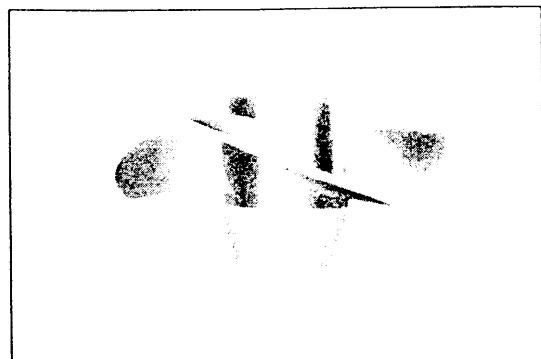
The turbine type may then be found by using the diagram, using the specific speed and the actual working head.



9: Diagram of specific speeds

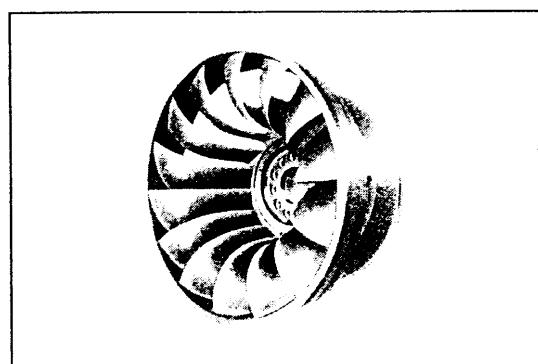
## 10. Diagram of turbine application ranges

The turbine type may be found by using the diagram, using the specific speed and the actual working head.



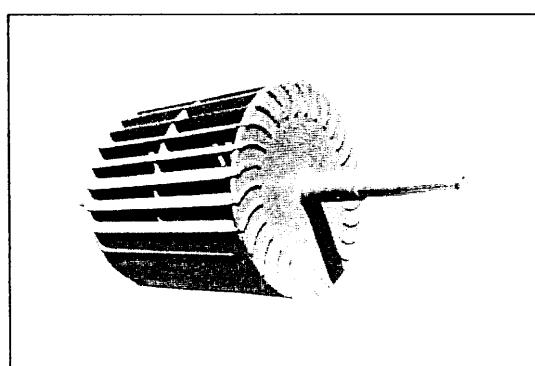
$n_s$ : 270 .... 1000  
 $N_s$ : 230 .... 860

Fig. 2.26: Kaplan/Propeller Runner



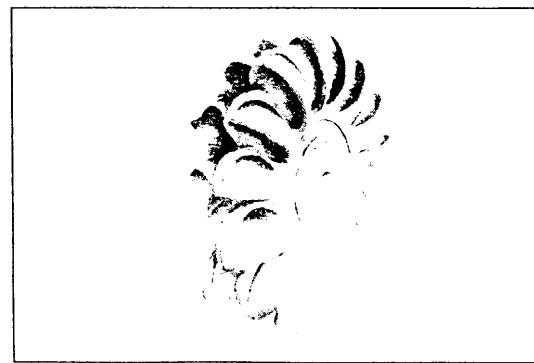
$n_s$ : 60 .... 350  
 $N_s$ : 50 .... 300

Fig. 2.27: Francis Runner



$n_s$ : 42 .... 170  
 $N_s$ : 36 .... 146

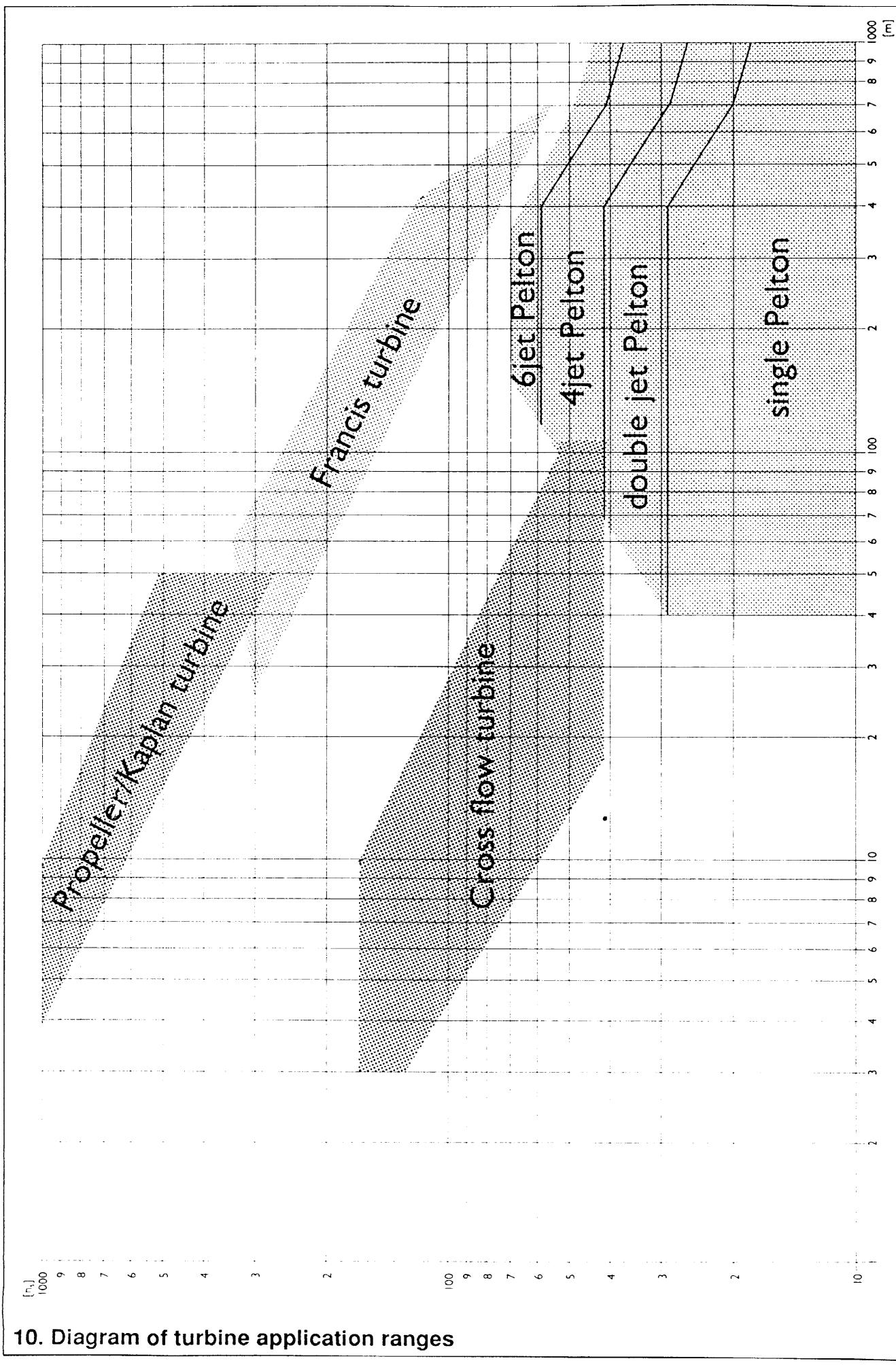
Fig. 2.28: Cross Flow Runner



$n_s$ : 8 .... 72  
 $N_s$ : 7 .... 62

Fig. 2.29: Pelton Runner

$$N_s = 0.86 \cdot n_s$$



10. Diagram of turbine application ranges

Harnessing Water Power on a Small Scale

Volume 2

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## Hydraulics Engineering Manual

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Supplement:

Selected  
Nomograms and Diagrams

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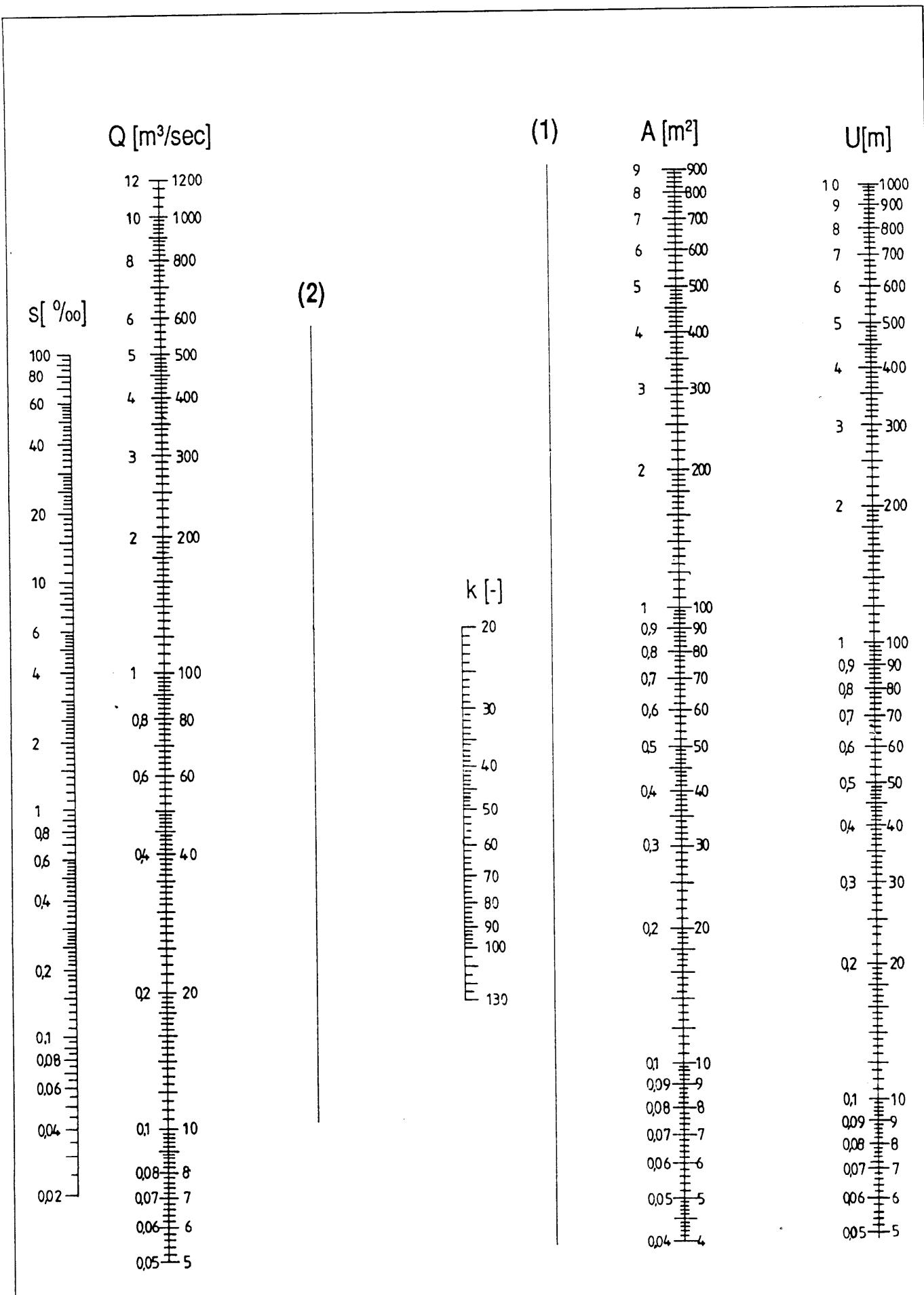
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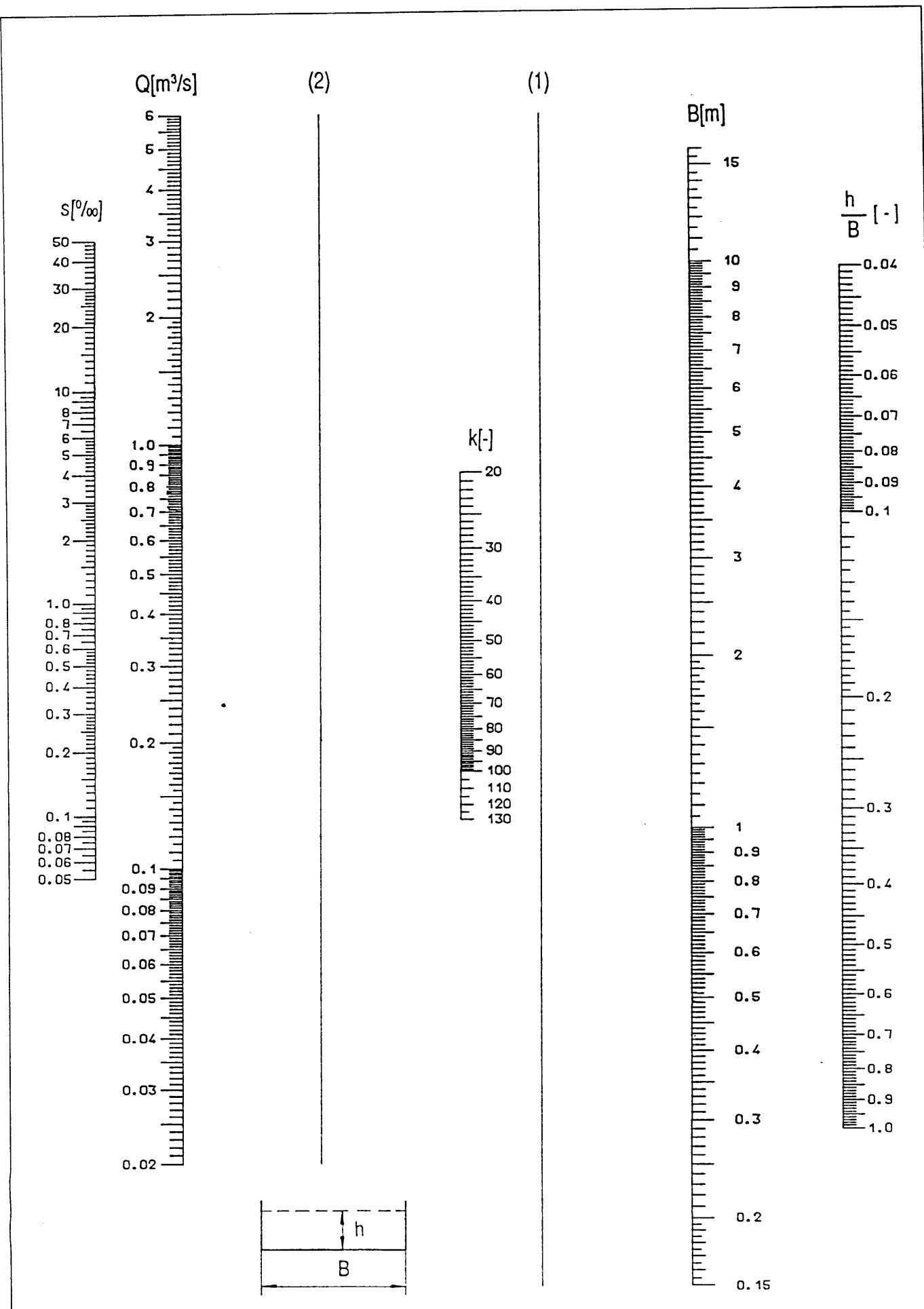
Selected Nomograms and Diagrams is a supplement to volume 2 of the series: Harnessing Water Power on a Small Scale, HYDRAULIC ENGINEERING MANUAL, by A. Arter and U. Meier, published by SKAT -Swiss Center for Appropriate Technology and available at:



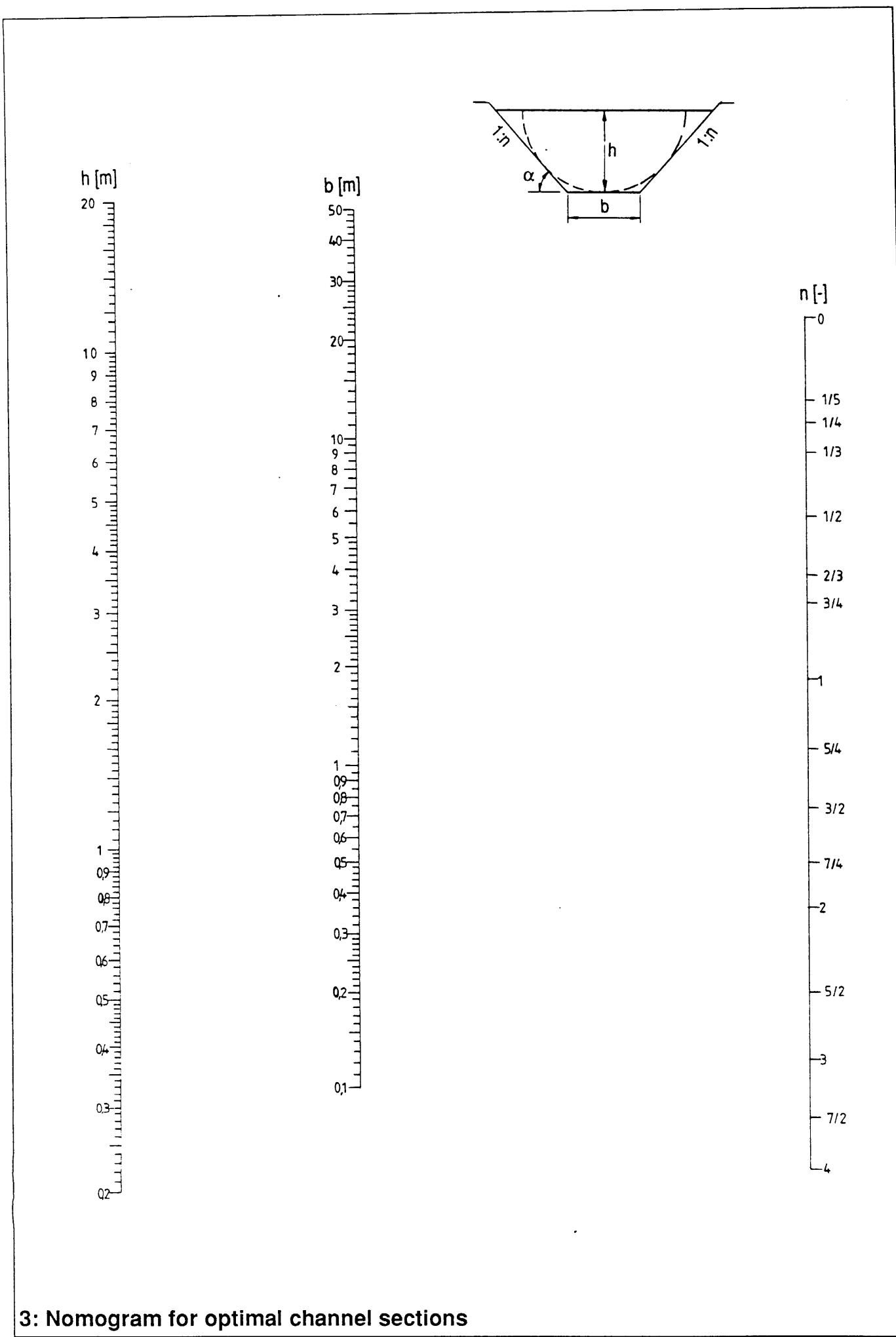
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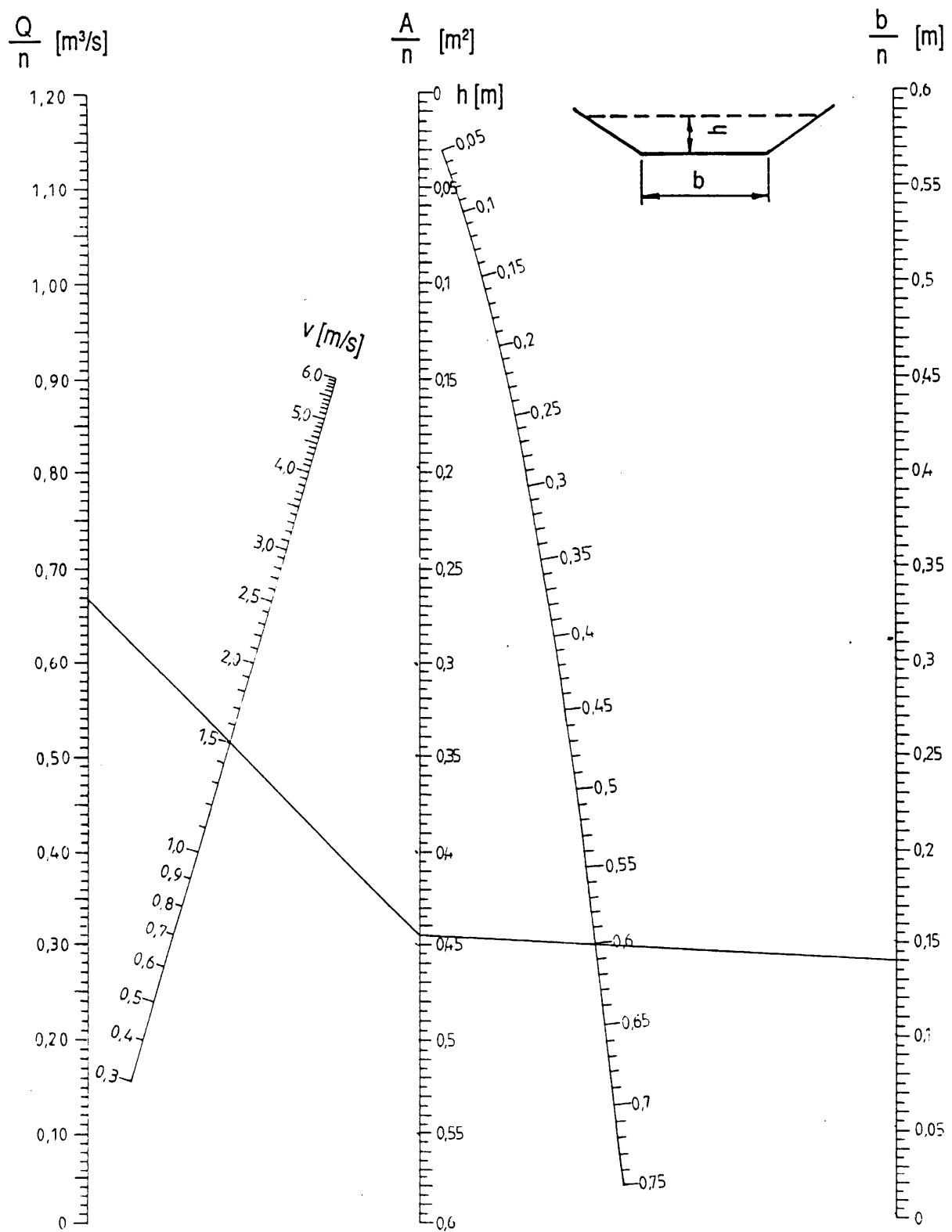
1: The STRICKLER Nomogram



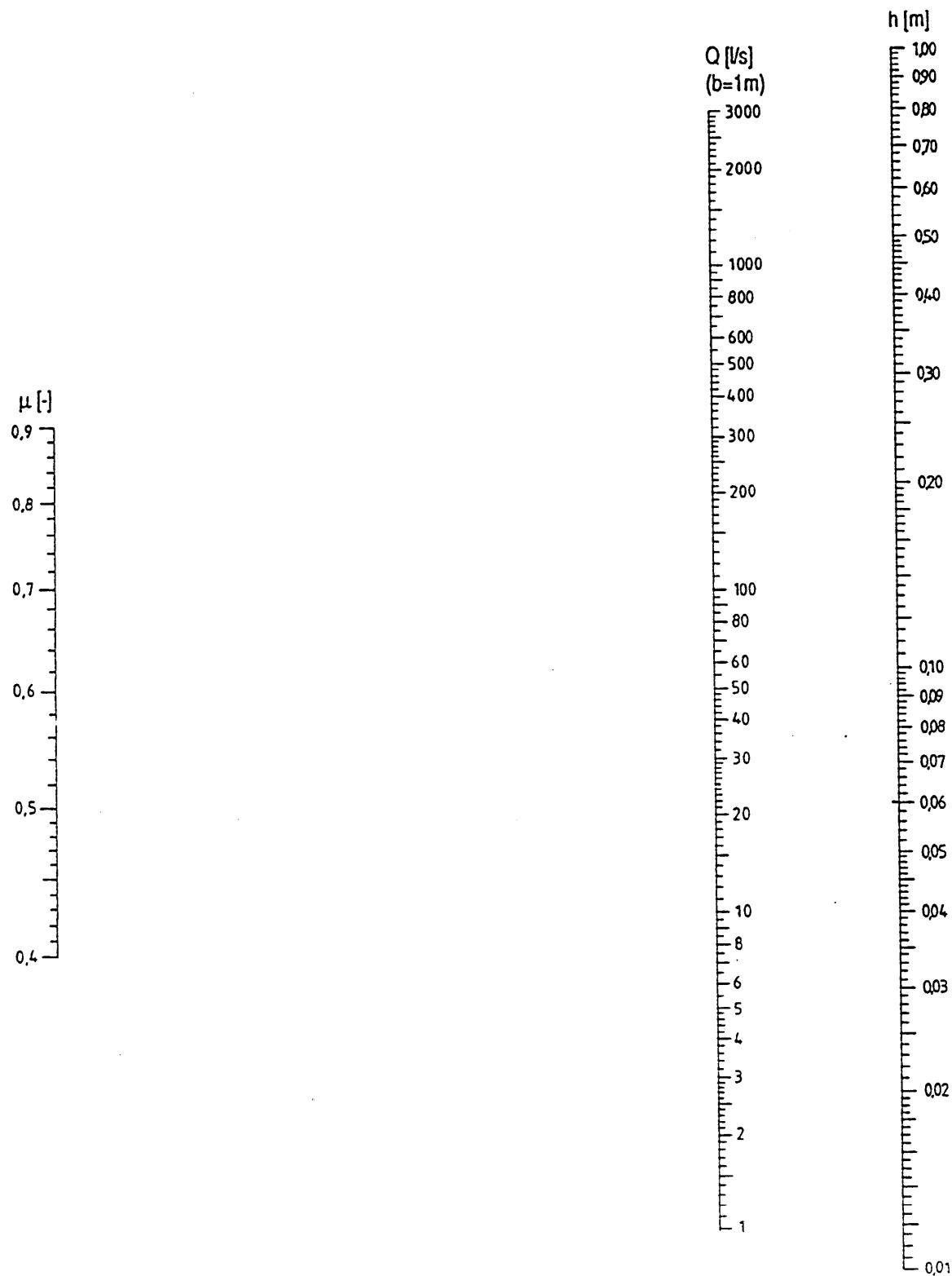
2: Nomogram for flow in rectangular canal cross sections



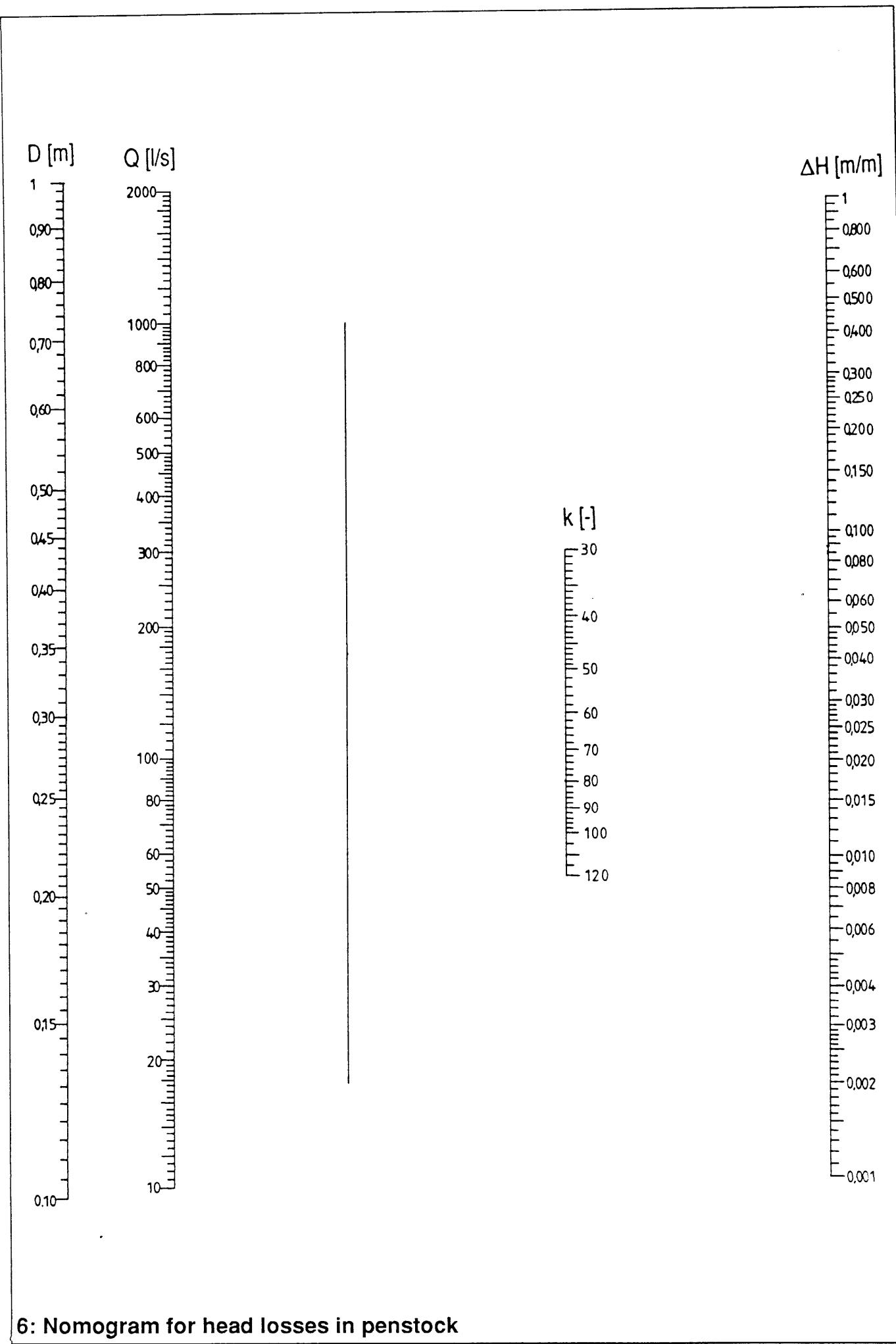
3: Nomogram for optimal channel sections



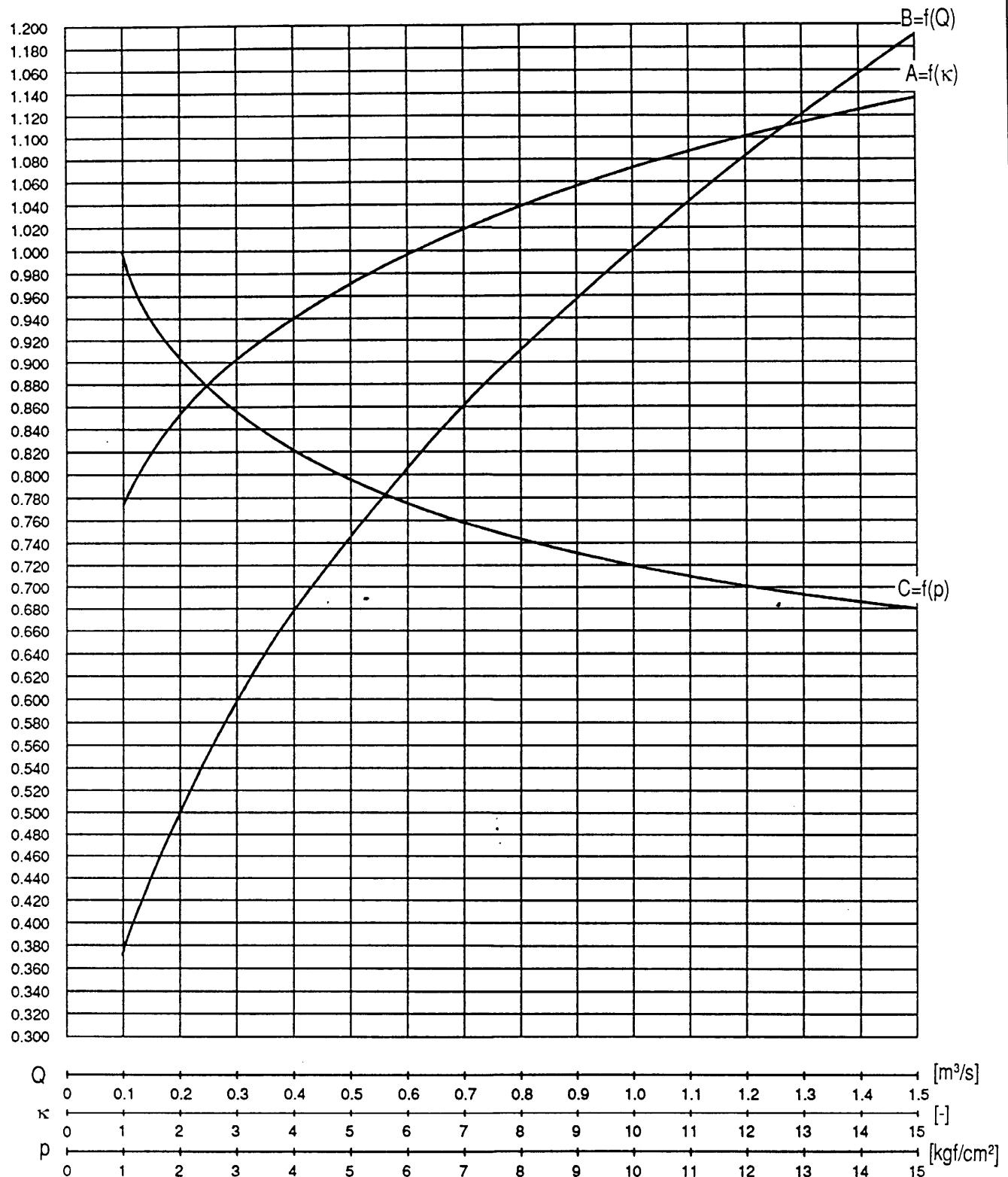
4: Nomogram for trapezoidal channel cross sections



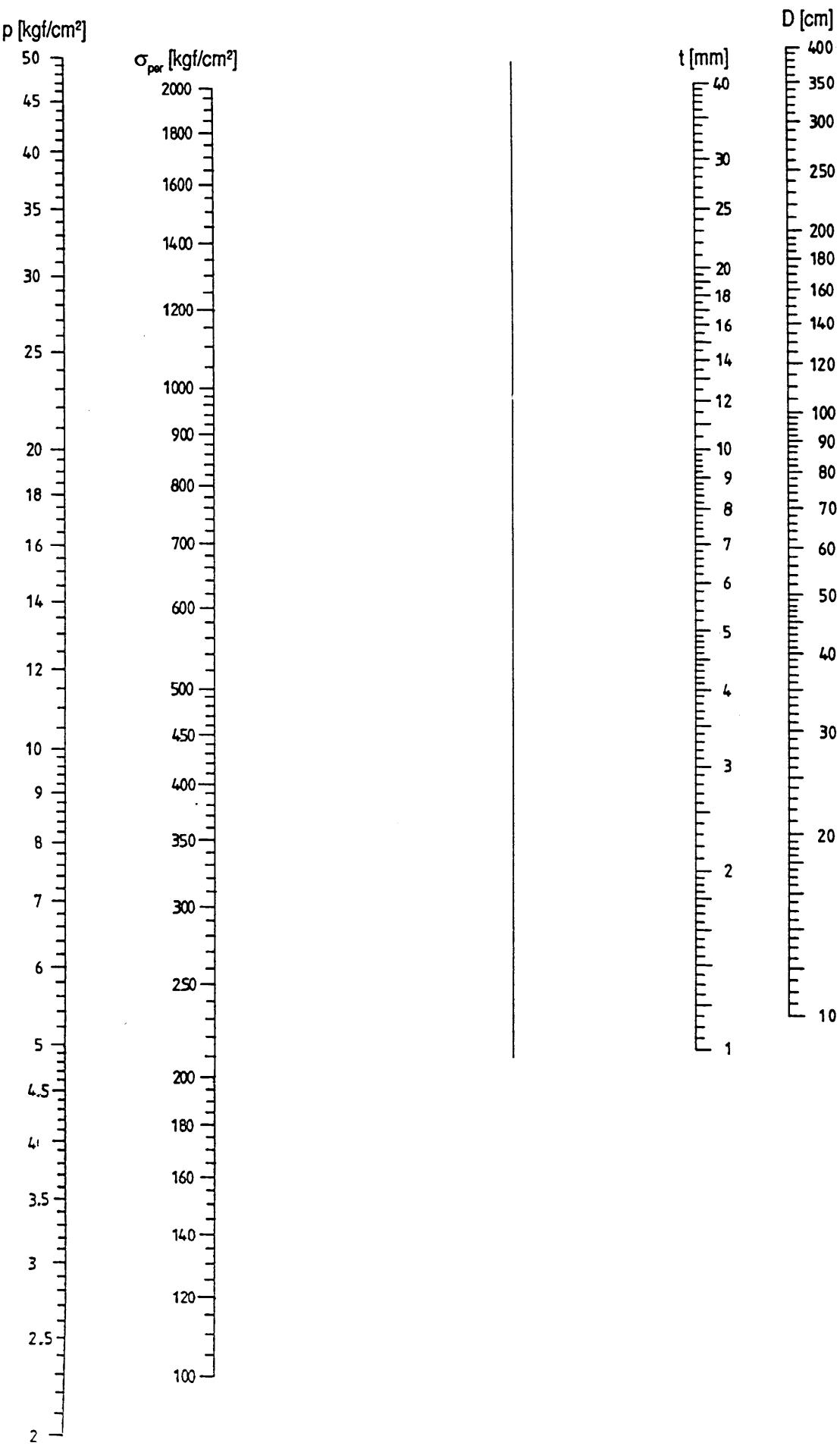
5: Nomogram for discharge measurement with weirs



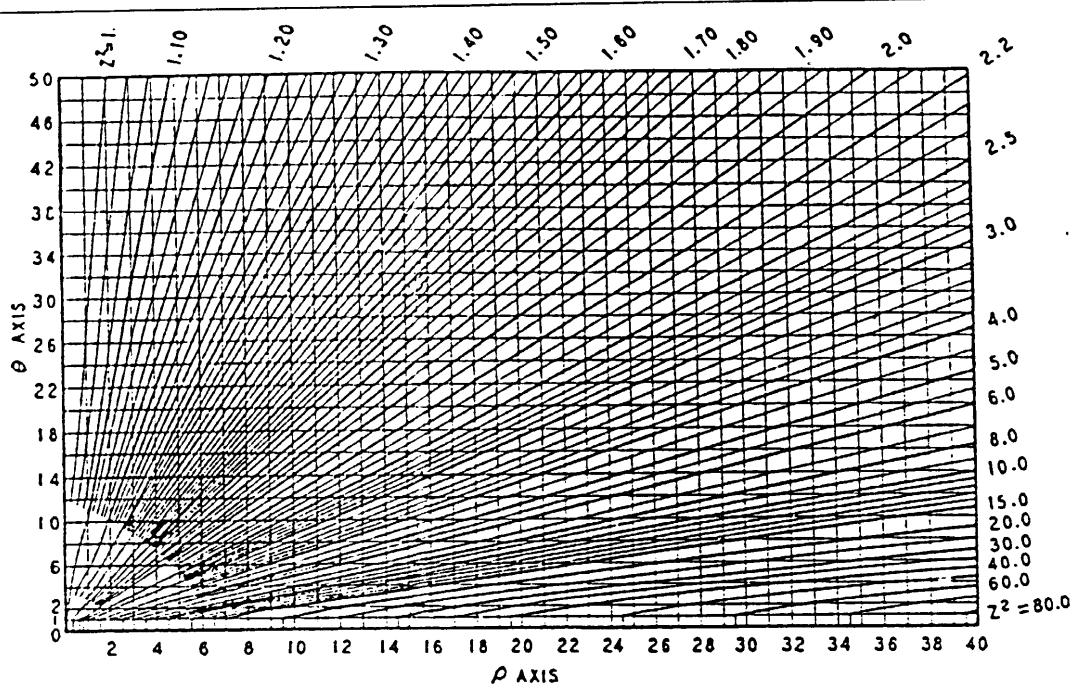
6: Nomogram for head losses in penstock



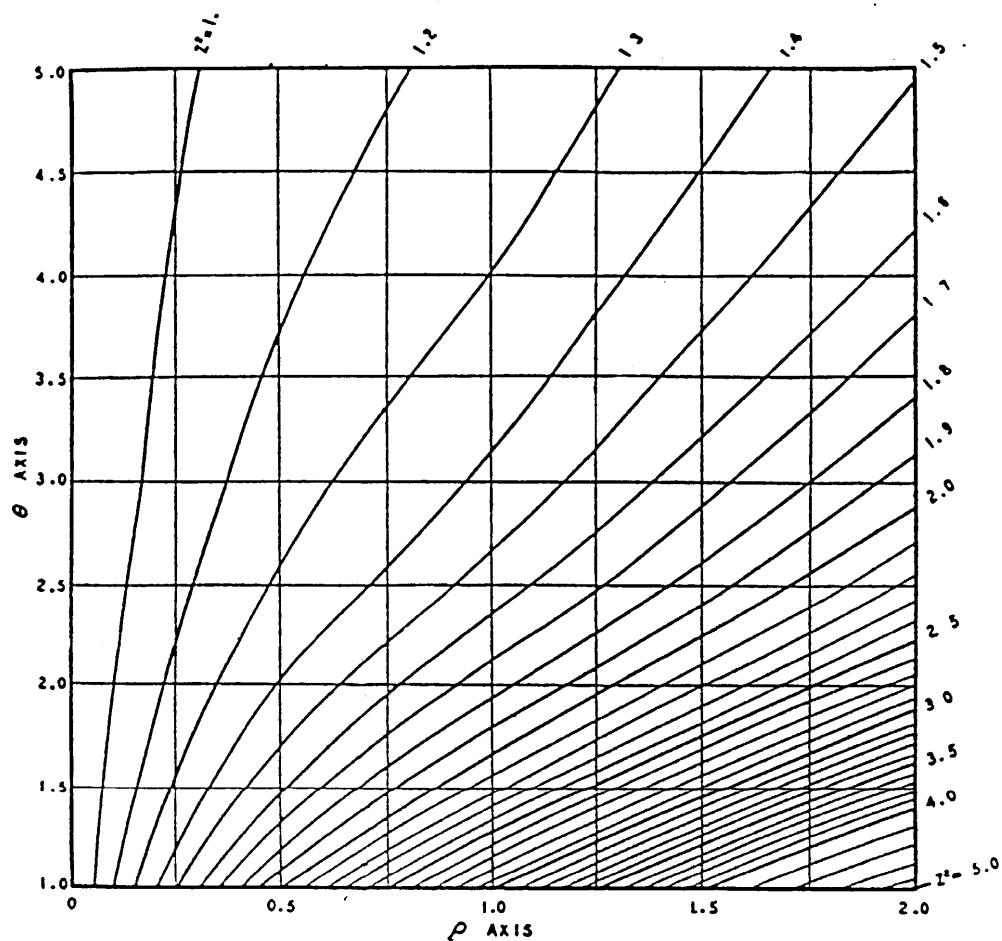
7: Diagram for the pre-selection of the economically optimal penstock diameter



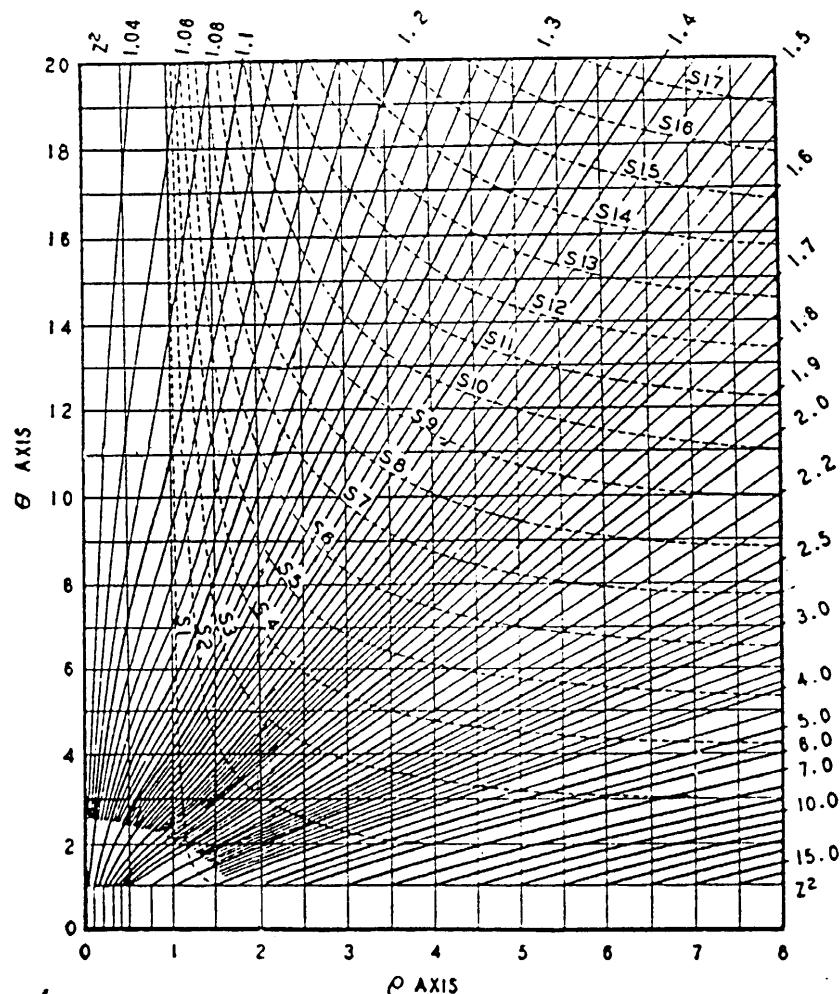
8: Nomogram for minimal penstock thickness



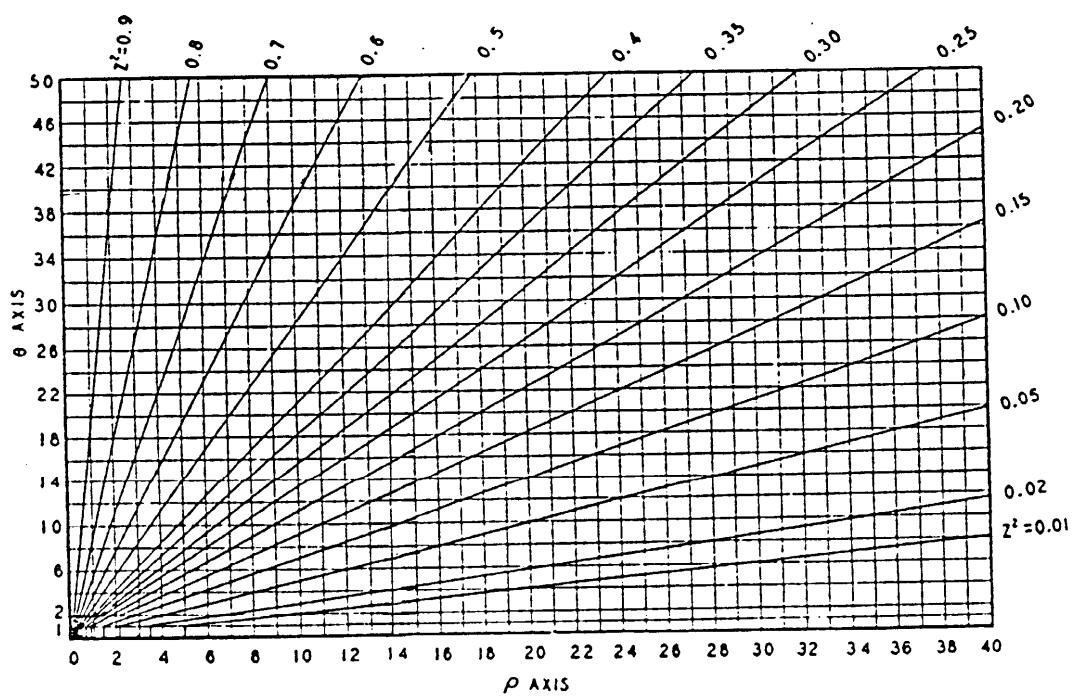
**9.1: ALLIEVI chart: pressure rise for uniform gate closure and simple conduits:  
large  $\rho$  and  $\theta$**



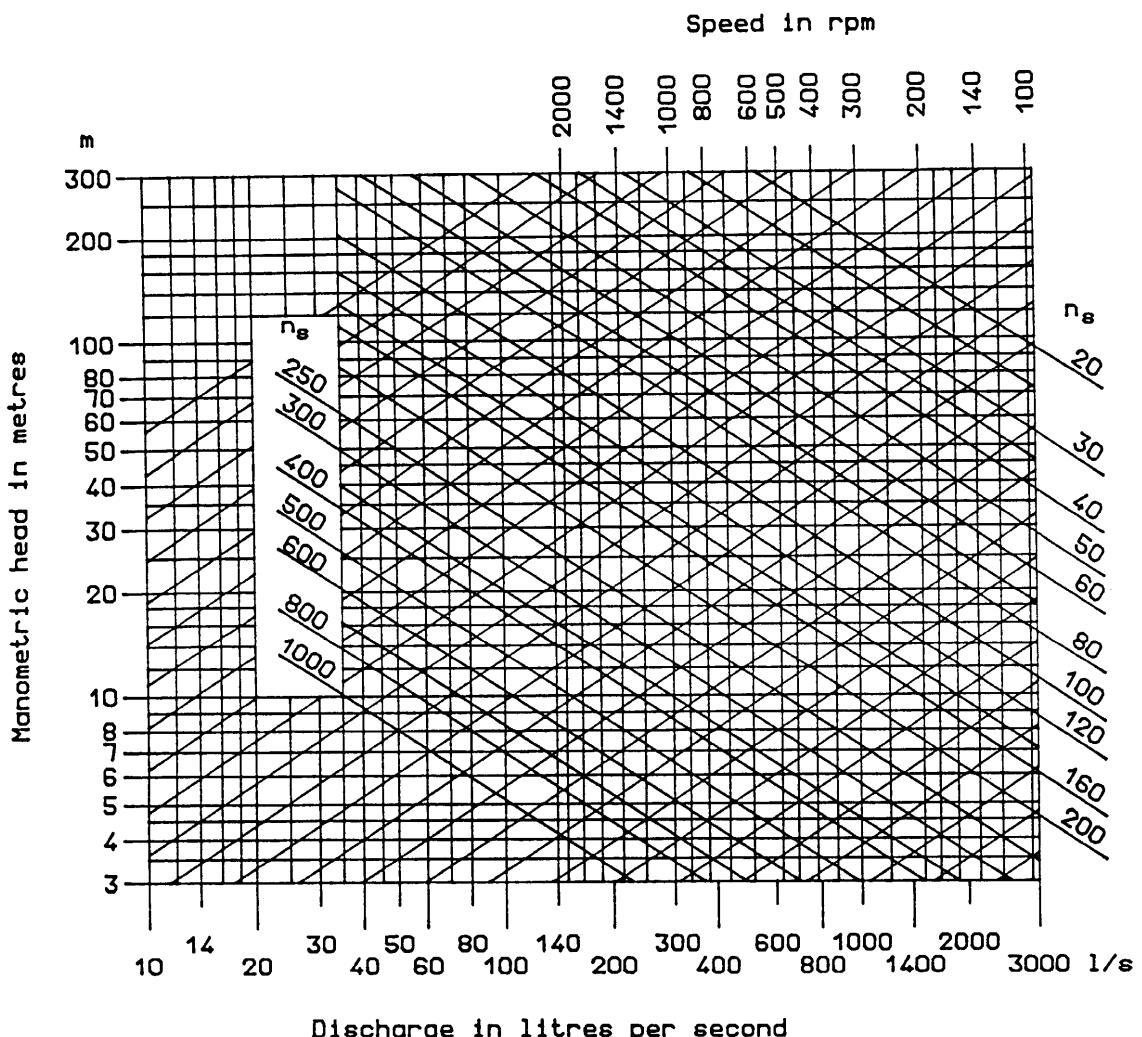
**9.2 :ALLIEVI chart: pressure rise for uniform gate closure and simple conduits:  
small  $\rho$  and  $\theta$**



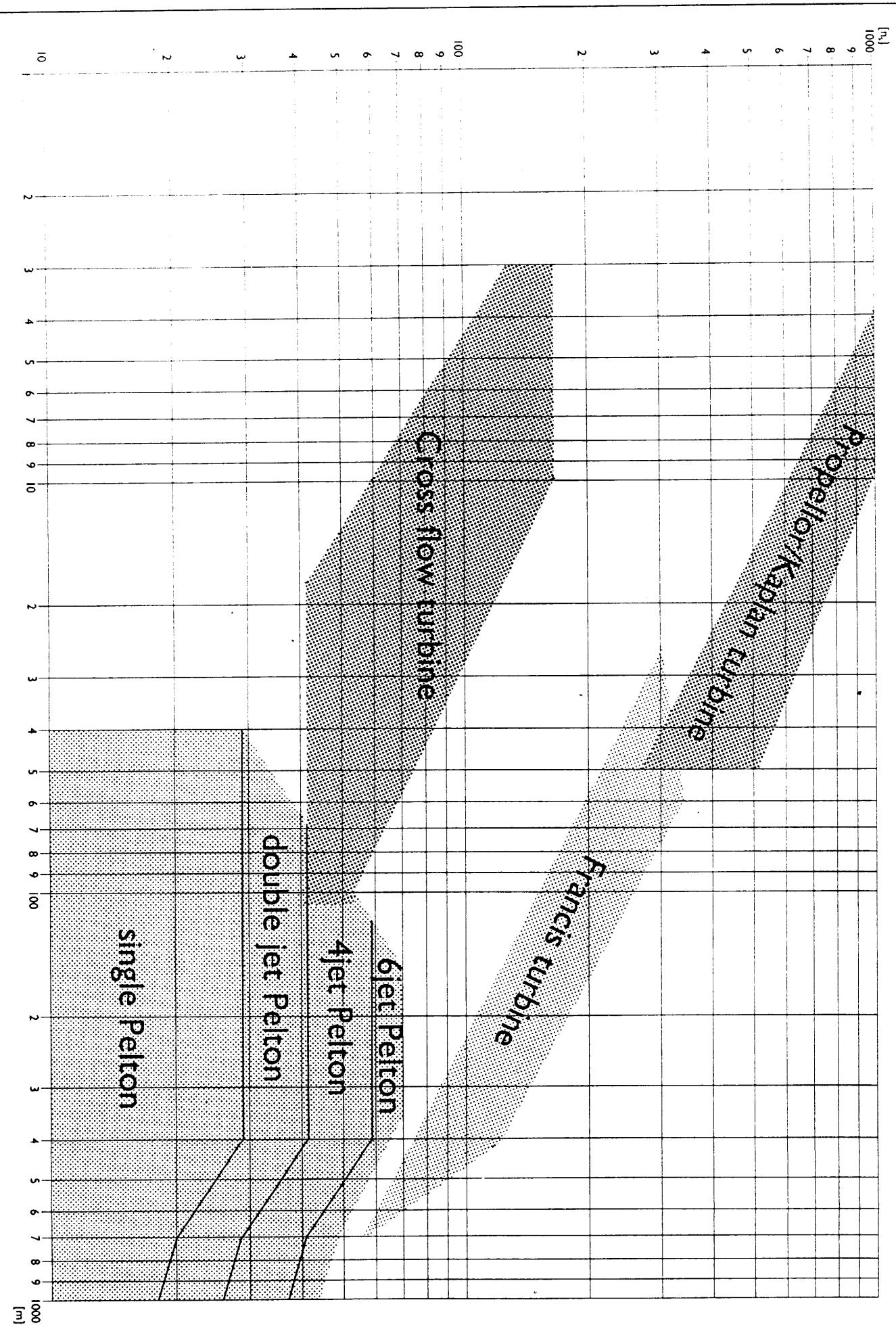
9.3 : ALLIEVI chart - pressure rise (medium  $\rho$  and  $\theta$ ) and wave cycle curves



9.4.: ALLIEVI chart: pressure drop



10: Diagram of specific speeds



11. Diagram of Turbine application ranges