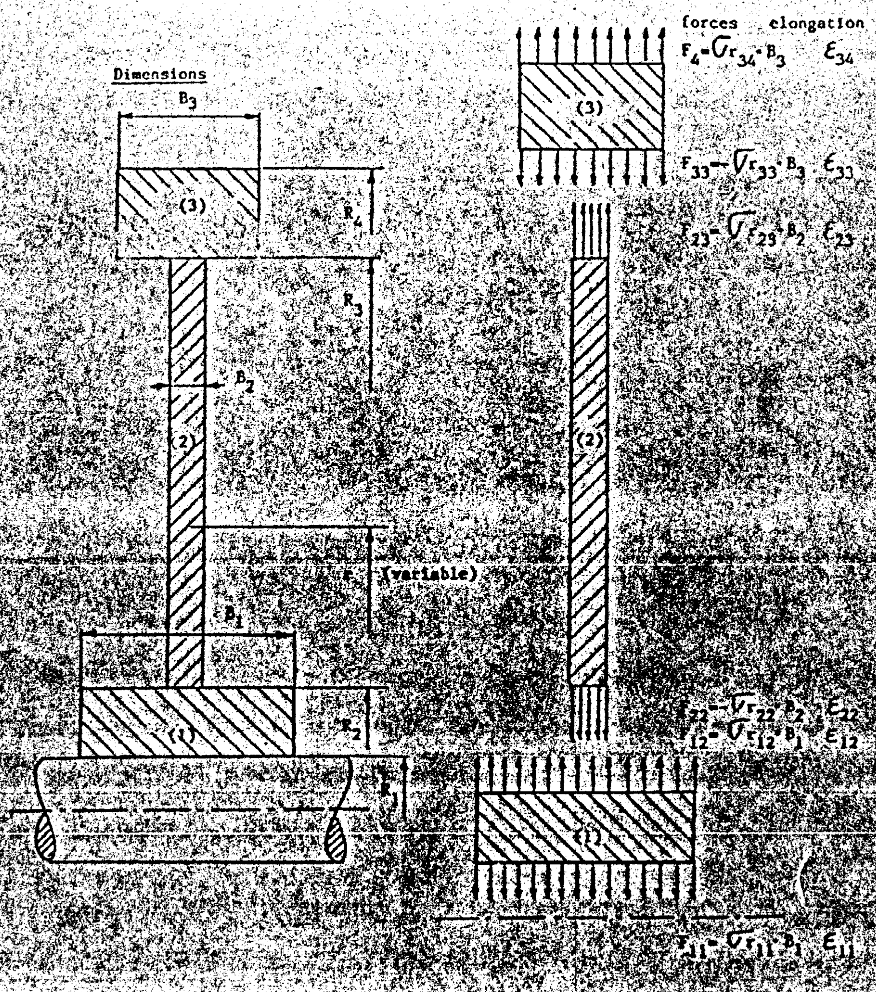
Working Paper
Document de Travail
Documento de Trabajo

07/87

Flywheel design for micro hydropower projects

Rim-type flywheel, dimensions and parameters



St.Gallen, August 87

SKAT

Varnbüelstraße 14, CH-9000 St.Gallen, Switzerland, Tel. 071 23 34 81

SKAT
Schweizerische Kontaktstelle für Angepasste Technik am ILE,
Institut für Lateinamerikaforschung und Entwicklungs-
zusammenarbeit an der Hochschule St.Gallen

SKAT
Centre Suisse pour la Technologie Appropriée à l'ILE,
Institut de Recherche sur l'Amérique Latine et de
Coopération au Développement, Université de Saint-Gall

SKAT
Swiss Center for Appropriate Technology at ILE,
Institute for Latin-American Research and for Development
Co-operation, University of Saint-Gall

SKAT
Centro Suizo de Tecnología Apropiada en el ILE,
Instituto de Investigación sobre América Latina y
de Cooperación al Desarrollo, Universidad de Sankt-Gallen

Jean-Marc Chapallaz
Ingénieur EPFL/SIA

FLYWHEEL DESIGN

Strength calculations and construction of a rim-type flywheel

Table of contents:

Vuiteboeuf, 28.8.1986

1. Introduction

The rim-type flywheel is made out of three parts:

- hub, with eventual keyslot or taper lock to hold the flywheel on the shaft
- outer rim of large section, thus concentrating the mass at the largest diameter for maximum inertia with minimum weight
- intermediate disk joining the two previous parts.

The rim-type flywheel may be casted or of welded construction.

2. Theory

2.1. Assumptions

The dimensions and other parameters are given on figure 2. For calculation purposes, the flywheel is separated into three partial disks (or rims), which are linked together through following boundary conditions:

a) radial forces are balanced
b) peripheral elongations are identical

No axial stress is considered in the calculation; that means the flywheel has a two dimensional configuration with tangential and radial stresses and strains.

A uniform longitudinal stress distribution is assumed. This may not be exactly true for the rim and the hub, if these are long and thin compared to the disk thickness, which induces then a fairly concentrated load (like a reinforcing ring on a cylindric pipe under pressure).

Notations:

R_{ij} = radius of element (i) at boundary (j)
B_i = width of element (i)
σr_{ij} = radial stress of element (i) at boundary (j)
σt_{ij} = tangential stress of element (i) at boundary (j)
εt_{ij} = tangential elongation of element (i) at boundary (j)
F_{ij} = $\sigma r_{ij} \times B_i$ = radial force acting at boundary (j) of element (i)

2.2. General stress formulae

The stresses in a rotating disk or ring of uniform thickness are given by following general formulae:

Radial stress

$$\sigma_r = \frac{E}{1-\nu^2}\left[C_1 \cdot (1+\nu) - C_2(1-\nu) \cdot \frac{1}{r^2}\right] - \frac{\rho \cdot \omega^2}{8}(3+\nu) \cdot r^2$$

Tangential stress

$$\sigma_t = \frac{E}{1-\nu^2}\left[C_1 \cdot (1+\nu) + C_2(1-\nu)\frac{1}{r^2}\right] - \frac{\rho \cdot \omega^2}{8} \cdot (1+3\nu) r^2$$

C_1 and C_2 are integration constants depending of the boundary conditions.

If $X = C_1 \cdot \frac{E}{\rho \omega^2} \cdot \frac{1+\nu}{1-\nu^2}$ $\quad Y = C_2 \cdot \frac{E}{\rho \omega^2} \cdot \frac{1-\nu}{1-\nu^2}$

$U = \frac{3+\nu}{8}$ $\quad V = \frac{1+3\nu}{8}$

The equations reduce to:

$$\frac{\sigma_r}{\rho \cdot \omega^2} = X - Y \cdot \frac{1}{r^2} - U \cdot r^2 \qquad (2.2.1)$$

$$\frac{\sigma_t}{\rho \cdot \omega^2} = X + Y \frac{1}{r^2} - V r^2 \qquad (2.2.2)$$

X, Y being the new integration constants
U, V constants

Units: X (m^2)
Y (m^4)
$\rho \cdot \omega^2$ (N/m^4)
σ (N/m^2)
U, V (-)

2.3. Boundary conditions

If we examine the boundary conditions between two contiguous elements (i) and (j) = (i+1), we find following relations:

Radial force balance:

$$F_{ij} = \sigma r_{ij} \cdot B_i \qquad F_{jj} = -\sigma r_{jj} \cdot B_j$$

and $F_{ij} + F_{jj} = 0$

thus $\sigma r_{ij} \cdot B_i = \sigma r_{jj} \cdot B_j$

Tangential elongation identity: $\varepsilon t_{ij} = \varepsilon t_{jj}$

the Hooks law states:

$$\varepsilon t \cdot E = \sigma t - \nu \cdot \sigma r$$

with E = elasticity modulus ν = Poisson's coefficient

= 2,1 10^{11} N/m^2 = 0,3 for steel

equalizing the elongations at the common boundary of element (i) and (j) = (i+1) gives

$$\sigma t_{ij} - \nu \cdot \sigma r_{ij} = \sigma t_{jj} - \nu \cdot \sigma r_{jj}$$

The equations (2.2.1) and (2.2.2) are used to calculate the integration constants X_i and Y_i of the element (i), X_j and Y_j of element (j) = (i+1).

Grouping the equations of all the elements produces a linear system, out of which the integration constants, thus the stress formulae, may be computed.

As example, a flywheel made out of 3 elements (hub - disk - rim) will be examined more accurately.

Boundary (1) - Shaft / hub:

The shaft or a taperlock induces a radial pressure P_1 on the hub

$$\sigma_{r11} = -P_1 \quad \text{thus} \quad X_1 - Y_1 \cdot \frac{1}{R_1^2} - UR_1^2 = -\frac{P_1}{\rho \cdot \omega^2}$$

$$\text{or} \quad \underline{X_1 - \frac{1}{R_1^2} \cdot Y_1 = UR_1^2 - \frac{P_1}{\rho \cdot \omega^2}}$$

Boundary (2) - Hub / disk:

Force balance:

$$\sigma_{r12} \cdot B_1 = \sigma_{r22} \cdot B_2$$

$$B_1(X_1 - Y_1 \frac{1}{R_2^2} - UR_2^2) - B_2(X_2 - Y_2 \frac{1}{R_2^2} - UR_2^2) = 0$$

or

$$B_1 \cdot X_1 - \frac{B_1}{R_2^2} \cdot Y_1 - B_2 \cdot X_2 + \frac{B_2}{R_2^2} \cdot \ldots Y_2 = (B_1 - B_2) \cdot UR_2^2$$

Tangential elongation:

$$\varepsilon_{t12} = \varepsilon_{t22} \quad \text{or} \quad \sigma_{t12} - \nu \sigma_{r12} = \sigma_{t22} - \nu \sigma_{r22}$$

$$X_1 + Y_1 \frac{1}{R_2^2} - VR_2^2 - \nu\left[X_1 - Y_1 \cdot \frac{1}{R_2^2} - UR_2^2\right] =$$

$$= X_2 + Y_2 \frac{1}{R_2^2} - VR_2^2 - \nu\left[X_2 - Y_2 \frac{1}{R_2^2} - UR_2^2\right]$$

$$\underline{(1-\nu) \cdot X_1 + \frac{1+\nu}{R_2^2} \cdot Y_1 - (1-\nu) \cdot X_2 - \left(\frac{1+\nu}{R_2^2}\right) \cdot Y_2 = 0}$$

Boundary (3) - Disk / rim

as for boundary (2), we obtain:

$$B_2 X_2 - \frac{B_2}{R_3^2} Y_1 - B_3 X_3 + \frac{B_3}{R_3^2} Y_3 = (B_2 - B_3) \cdot U R_3^2$$

and

$$(1-\nu) X_2 + \frac{1+\nu}{R_3^2} \cdot Y_2 - (1-\nu) \cdot X_3 - \left(\frac{1+\nu}{R_3^2}\right) \cdot Y_3 = 0$$

Boundary (4) - Outer surface of rim

No radial force or pressure is assumed acting on the outer

No radial force or pressure is assumed acting on the outer
surface of the rim:

$$\sigma_{r34} = 0$$

and $$X_3 - \frac{1}{R_4^2} \cdot Y_3 - U R_4^2 = 0$$

$$X_3 - \frac{1}{R_4^2} \cdot Y_3 = U R_4^2$$

With the help of these 6 boundary equations or conditions, it is now possible to compute the value of the integration constants, and to calculate the stresses in the flywheel

2.4. Computation of the integration constants

The six equations give following linear system:

$$(1)\quad a_{11}x_1 + a_{12}x_2 = b_1$$

$$(2)\quad a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2$$

$$(3)\quad a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = b_3$$

$$(4)\quad a_{43}x_3 + a_{44}x_4 + a_{45}x_5 + a_{46}x_6 = b_4$$

$$(5)\quad a_{53}x_3 + a_{54}x_4 + a_{55}x_5 + a_{56}x_6 = b_5$$

$$(6)\quad a_{65}x_5 + a_{66}x_6 = b_6$$

with:

$x_1 = X_1 \quad x_2 = Y_1 \quad x_3 = X_2 \quad x_4 = Y_2 \quad x_5 = X_3 \quad x_6 = Y_3$

$a_{11} = 1 \quad a_{12} = -\frac{1}{R_1^2}$

$a_{21} = B_1 \quad a_{22} = -\frac{B_1}{R_2^2} \quad a_{23} = -B_2 \quad a_{24} = +\frac{B_2}{R_2^2}$

$a_{31} = 1-\nu \quad a_{32} = \frac{1+\nu}{R_2^2} \quad a_{33} = -(1-\nu) \quad a_{34} = -\frac{1+\nu}{R_2^2}$

$a_{43} = 1-\nu \quad a_{44} = \frac{1+\nu}{R_3^2} \quad a_{45} = -(1-\nu) \quad a_{46} = -\frac{1+\nu}{R_3^2}$

$a_{53} = B_2 \quad a_{54} = -\frac{B_2}{R_3^2} \quad a_{55} = -B_3 \quad a_{56} = +\frac{B_3}{R_3^2}$

$a_{65} = 1 \quad a_{66} = -\frac{1}{R_4^2}$

$b_1 = UR_1^2 - \frac{P_1}{\rho \cdot \omega^2} \quad b_2 = (B_1 - B_2)UR_2^2 \quad b_3 = 0$

$b_4 = 0 \quad b_5 = (B_2 - B_3)\cdot UR_3^2 \quad b_6 = UR_4^2$

The system may be also represented in matricial form:

$$\begin{bmatrix} 1 & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & a_{46} \\ 0 & 0 & a_{53} & a_{54} & a_{55} & a_{55} \\ 0 & 0 & 0 & 0 & 1 & a_{66} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix}$$

Computation:

The solution of these simultaneous linear equations may be obtained manually, for example through a method based on elimination, or with the help of a computer.
One way is to transform the coefficiant matrix system into the form:

$$\begin{bmatrix} 1 & m_{12} & 0 & 0 & 0 & 0 \\ 0 & 1,0 & m_{23} & m_{24} & 0 & 0 \\ 0 & 0 & 1,0 & m_{34} & 0 & 0 \\ 0 & 0 & 0 & 1,0 & m_{45} & m_{46} \\ 0 & 0 & 0 & 0 & 1,0 & m_{56} \\ 0 & 0 & 0 & 0 & 0 & 1,0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{bmatrix}$$

m_{ij} and p_i are the new constants computed through elimination by substracting one equation from another, multiplyed by a suitable coefficiant, to obtain coefficiant 1 on the diagonal of the matrix.

3. Application - Example of calculation

3.1. Datas and parameters

A flywheel made according to figure 3.1. will be calculated. Its characteristics are:

Dimensions:

R_1 = 31,5 mm = 0,0315 m B_1 = 100 mm = 0,1 m

R_2 = 60 mm = 0,060 m B_2 = 20 mm = 0,02 m

R_3 = 315 mm = 0,315 m B_3 = 56 mm = 0,056m

R_4 = 375 mm = 0,375 m

Rotating speed: n = 2860 rpm

ω = 300 rad/s

Specific mass ρ = 7800 kg/m^3

Parameter:

Poisson's coefficiant: ν = 0,3

Stress equations constants:

$$U = \frac{3+\nu}{8} = 0,4125$$

$$V = \frac{1+3\nu}{8} = 0,2375$$

$$\rho\omega^2 = 7,02\ 10^8\ (N/m^4)$$

3.2. Method and assumptions

The flywheel taken as an example is of welded construction, the welding being made at the boundaries hub-disk and disk-rim.

Using the theoretical relations given in chapter 2, three configurations will be computed, and their results compared:

3.3.: complete homogeneous flywheel (hub+disk+rim)
3.4.: partial flywheel, including rim and disk, without hub
3.5.: all three elements (hub, disk, rim) taken separately as independantly rotating bodies (unbounded)

At that stage, the local influence of the welding as well as of non-uniform longitudinal stress distribution are not taken into account.

3.3. Stresses in the homogeneous rim-type flywheel

Using the formulae given under 2.4. the integration constants will first be computed out of the linear equations system. The stresses are then calculated using the formulae given under 2.2.

Constants:

$$x_1 = X_1 = 29,3436\ 10^{-3}$$
$$x_2 = Y_1 = 28,710\ 10^{-6}$$
$$x_3 = X_2 = 81,0405\ 10^{-3}$$
$$x_4 = Y_2 = -71,5025\ 10^{-6}$$
$$x_5 = X_3 = 63,9825\ 10^{-3}$$
$$x_6 = Y_3 = 0,840203\ 10^{-3}$$

Radial stress: $\sigma_{r_i} = \rho \cdot \omega^2 (X_i - \frac{Y_i}{r^2} - U \cdot r^2)$ (2.2.1.)

Tangential stress: $\sigma_{t_i} = \rho \cdot \omega^2 (X_i + \frac{Y_i}{r^2} - V \cdot r^2)$ (2.2.2.)

Resultant stress: $\sigma_i = \sqrt{\sigma_r^2 + \sigma_t^2}$

with (i) nr. of the concerned element of the flywheel.

The stress distributions with the radius are represented on figure 3.3.

Observation:

The calculation shows that the critical section is at the boundary hub-disk, the stresses decreasing from that spot with increasing radius.

3.4. Stresses in the partial flywheel disk+rim

The boundary conditions presented for a tree-element flywheel may be applied also for the two-element system. The linear system reduces to four equations giving four integration constants.

These are:

$$x_1 = X_2 = 82,1578 \quad 10^{-3}$$
$$x_2 = Y_2 = 290,4217 \quad 10^{-3}$$
$$x_3 = X_3 = 66,15386 \quad 10^{-3}$$
$$x_4 = Y_3 = 1,14554 \quad 10^{-3}$$

The stress distribution, computed with the help of equations 2.2.1. and 2.2.2., is represented graphically on figure 3.4.

Observation:

The critical section remains at the inner diameter of the disk, with increased stress values compared to those found in the complete flywheel (+40%).
These are the maximum possible, after failure or plastic deformation of the weld bead.

The distribution of stress at larger radii does not show a spectacular increase of these.

3.5. Stresses in rim, disk and hub considered as independant elements

The elements rotating independantly from each other may be calculated with the formulae used for a homogeneous circular disk with a central hole or by computing the integration constants of the general equations.

In that case, we obtain:

rim alone: $x_1 = X_3 = 98{,}938 \quad 10^{-3}$

$x_2 = Y_3 = 5{,}7558 \quad 10^{-3}$

disk alone: $x_1 = X_2 = 42{,}4153 \quad 10^{-3}$

$x_2 = Y_2 = 0{,}147349 \quad 10^{-3}$

hub alone: $x_1 = X_1 = 1{,}8943 \quad 10^{-3}$

$x_2 = Y_1 = 1{,}4735 \quad 10^{-6}$

The calculated stress distributions are represented on figure 3.5.

Observation:

The stress in the rim is the highest, with a similar value to the one at the critical section of the complete fly-wheel, that means close to the admissible stress.

Reinforcing the rim with the disk reduces its stress by more than twice, but transfers the maximum sollicitation into the latter.

4. Discussion

Welding:

The calculations show that the critical section lies at the boundary between disk and hub, where the two parts are welded together.

Therefore: the admissible stress adopted should take into account the quality of the welding which induces:

* local material quality and strength variation
* local stress concentration due to surface irregularities (at junction of weld bead with element)
* local stress concentration due to partially welded boundary (remaining gap between the two weld bead, see figure 3.1.)

Proposal for a safe dimensioning:

Assuming that the welding quality may not be guaranteed and checked, the flywheel may be designed to be as far as possible fail safe.

Following way is possible:

* dimension the rim as an independant rotating ring according to the admissible stress (see 3.5.)
* dimension the rim+disk system so that the stress at the boundary disk-hub remains admissible (see 3.4.)
* check the stress distribution in the whole system, taking into account a welding and a stress concentration factor (see 3.3.)

In that way, a failure or a plastic deformation at the welded boundaries should not lead to the collapse of the whole flywheel.

References - Literature

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Rim-type flywheel, dimensions and parameters

Figure 2

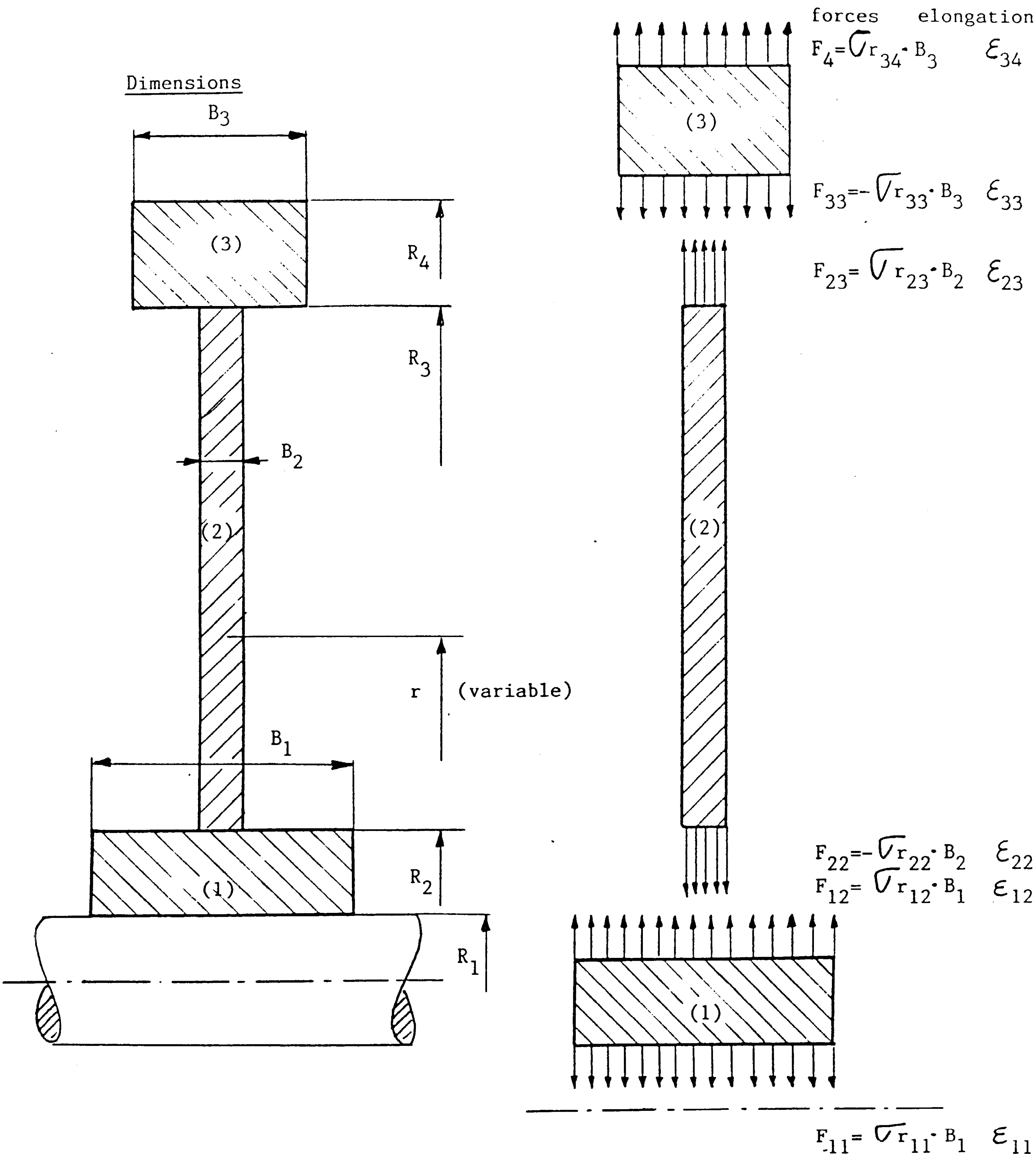
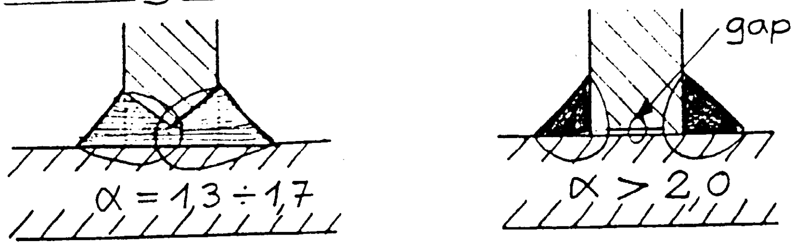


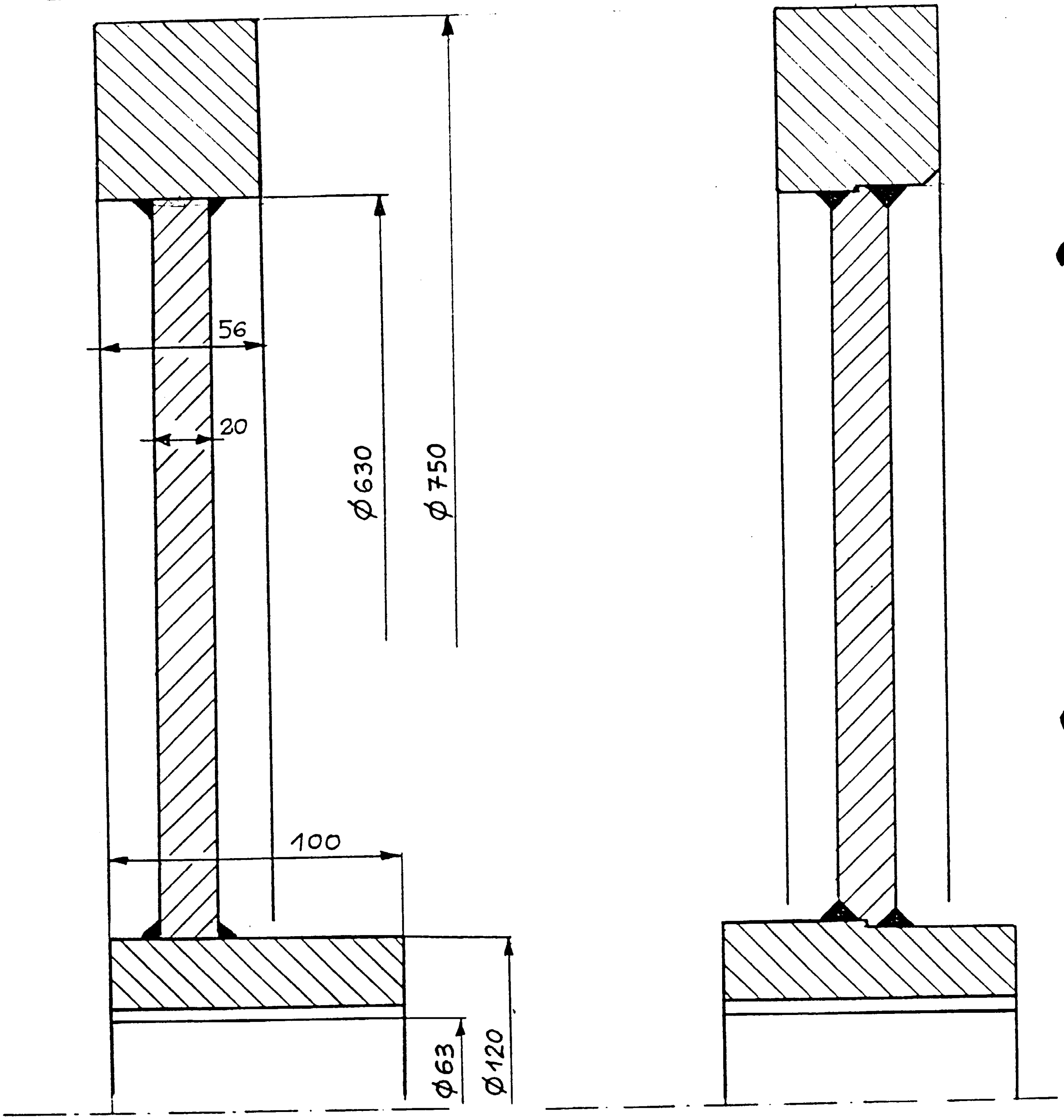
Figure 3.1

Welding details:



α = stress concentration factor

Rim-type flywheel design – Example of calculation



Initial design | Proposed improved design

SYANGJA Flywheel

diam. 750 mm

$GD^2 = 40\ kgm^2$

Weight : 105 kg

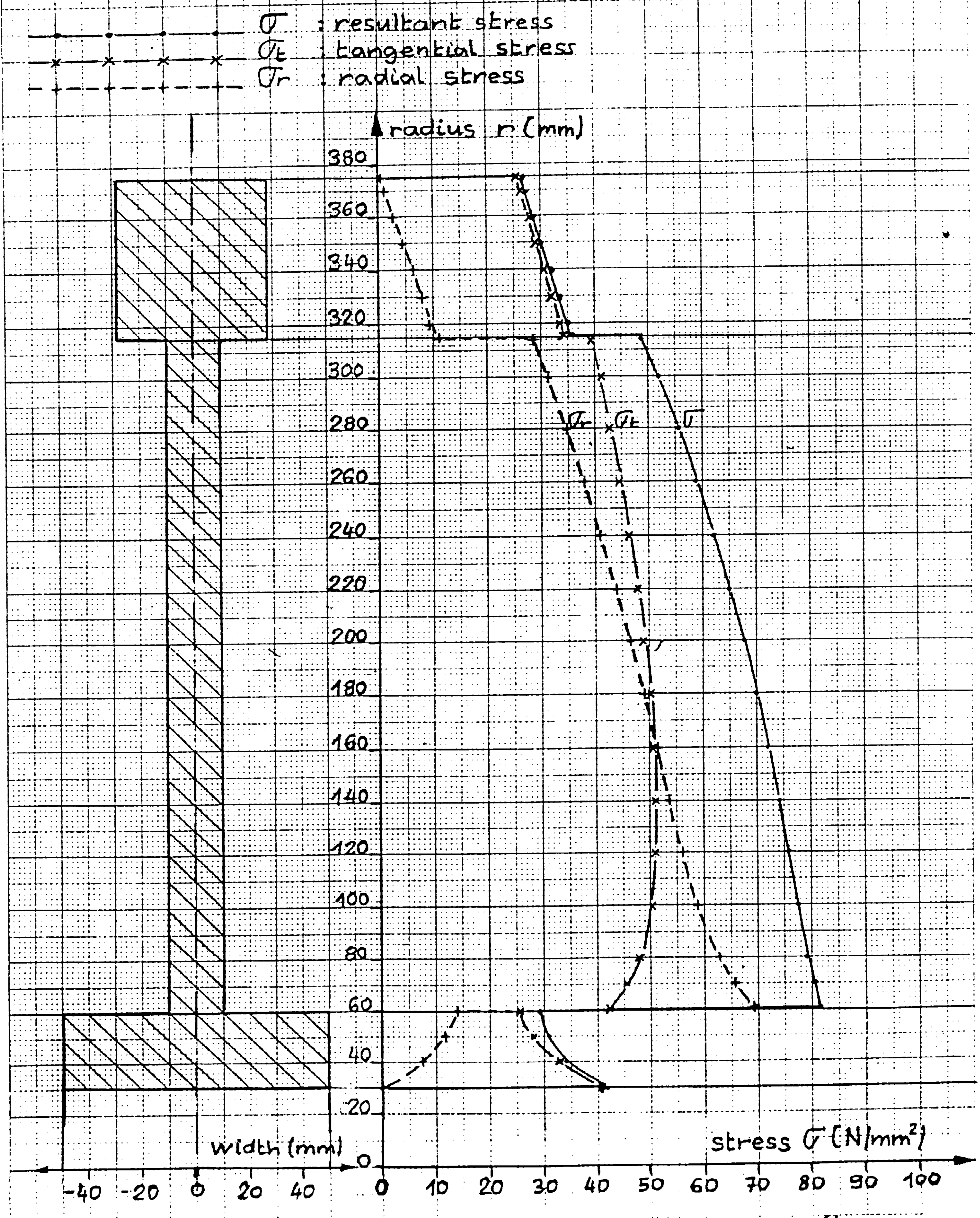
Calculation of a rim-type flywheel

Figure 3.3

Theoretical stress distribution

System considered: homogeneous flywheel [hub + disk + rim]

Rotating speed: $\omega = 300$ rad/s Material: steel St 37



22.8.86

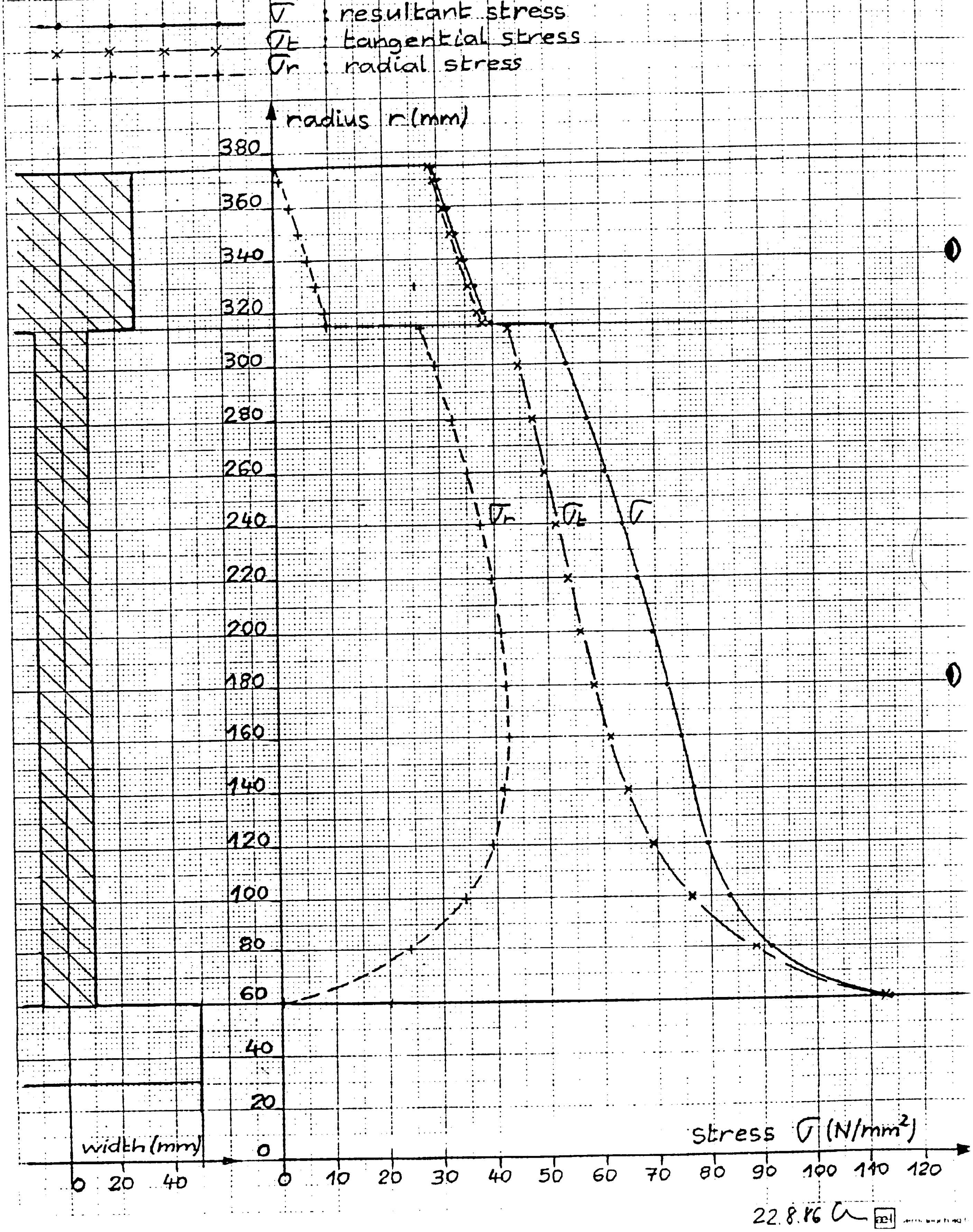
Figure 3.4.

Calculation of a rim-type flywheel

Theoretical stress distribution

System considered: partial hom. flywheel (disk + rim, without hub)

Rotating speed: $\omega = 300$ rad/s   Material: steel St 37



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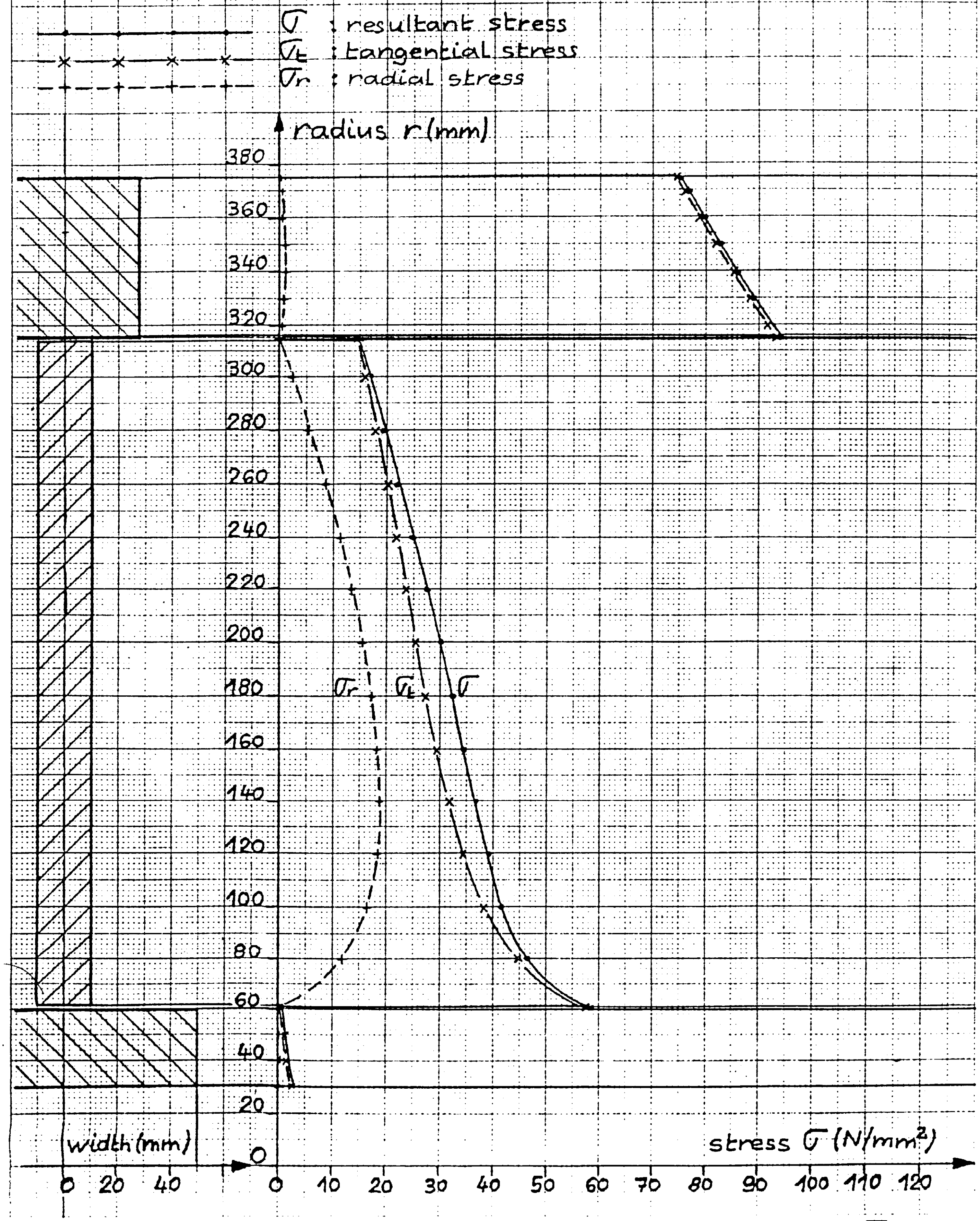
Calculation of a rim-type flywheel

Figure 3.5.

Theoretical stress distribution

System considered: rim, disk, hub as independant elements (unbounded)

Rotating speed: ω = 300 rad/s   Material: steel St 37



Project 0520 - Flywheel design

Disk-type flywheel - Example of calculation

According to drawing 86 - SP/13 - 16/a

Stress

Formulas for solid homogeneous circular disk of uniform thickness with a central hole are used.

Diameter: D = 1000 mm d = 95 mm

Radius: R = 0,50 m R_o = 0,0475 m $\frac{R_o}{R} = 0{,}10$

Rotation speed assumed: n = 1500 x 1,8 = 2700 rpm
= 283 rad/s

Maximum stress at central hole:

tangential stress = resultant stress: $\sigma_t = 0{,}8268 \cdot \rho \cdot \omega^2 R^2$
$\sigma_t = 129\ N/mm^2$

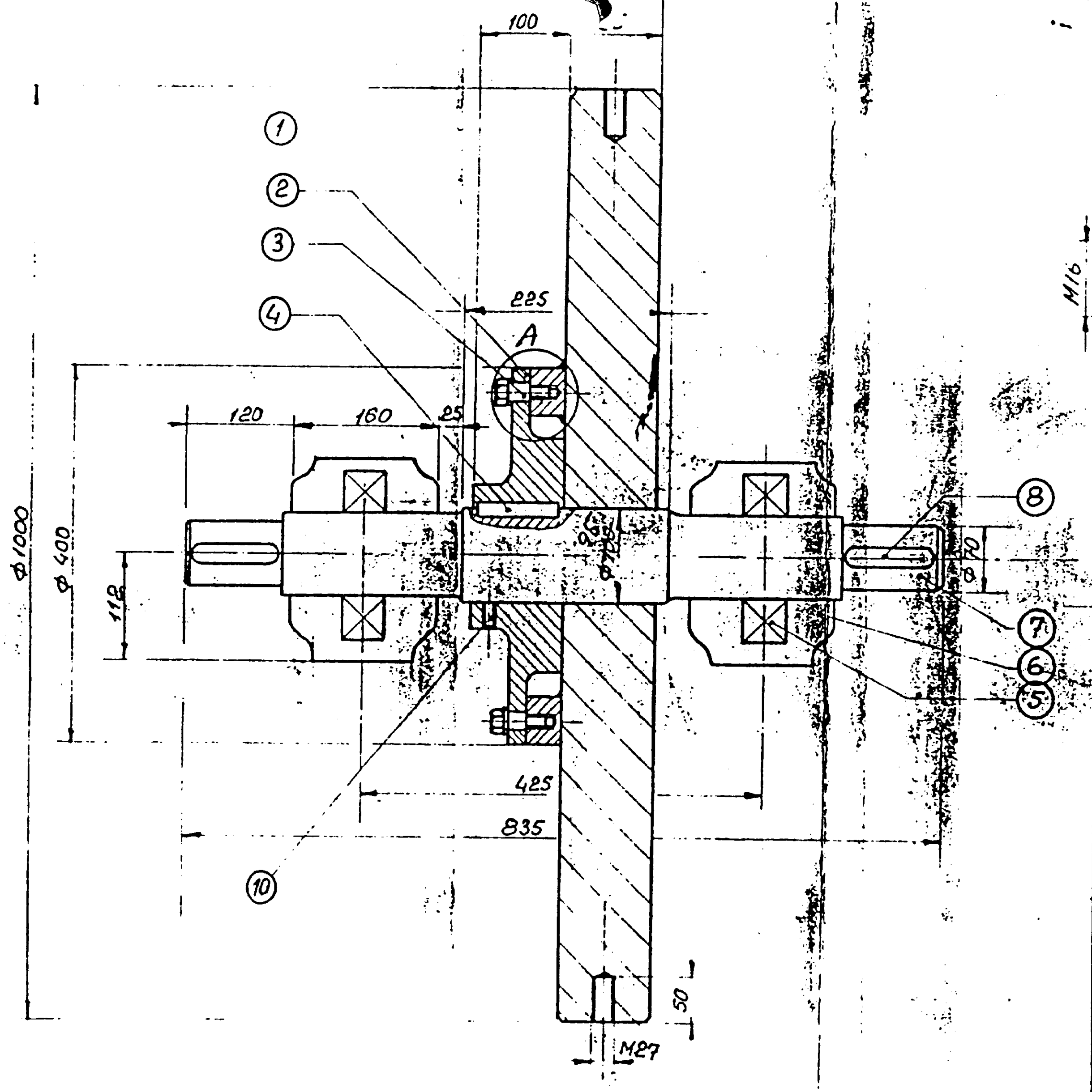
Safety factor: $s = \frac{\sigma_y}{\sigma_t} = \frac{240}{129} = 1{,}86$

with $\sigma_y = 240\ N/mm^2$ = yield stress of steel St 37

The disk is not weakened through fixation holes or key slot, so that this stress is admissible. It is assumed that:

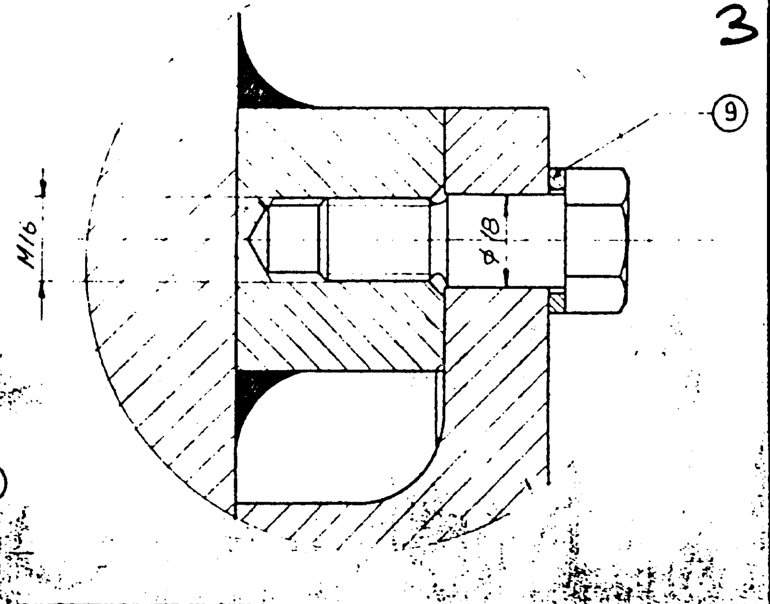
- the flywheel is well balanced, statically and dynamically *
- the internal residual stresses are relieved
- both faces are machined and are smooth, also at the weld bead of the fixing ring, with no surface irregularities (no cracks)
- the material is homogeneous and its mechanical properties are guaranteed
- assembly flywheel-shaft without play.

* dynamic forces are not only induced by unbalance but also by gyroscopic effect occuring if the disk is not truly perpendicular to its rotation axis.



Detail A

3



Pos.No	Description	Refer to Drg. No.	Sheet
10	Socket screw		
9	Spring Washer	86-SP/13-16/a-9	
8	Key	86-SP/13-16/a-8	
7	Fly Wheel Shaft	86-SP/13-16/a-7	A4
6	Bearing Housing	SKF SNA 520 TC	
5	Bearing	SKF 2220 K	
4	Key	86-SP/13-16/a-4	
3	Stud	86-SP/13-16/a-3	
2	Fly Wheel Hub Flange	86-SP/13-16/a-2	A3
1	Fly Wheel	86-SP/13-16/a-1	A3

Fly Wheel Assembly	
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